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Homework on Mixed Strategies

Exercise 1: (Final 2018) Find all the Nash equilibria (in pure and/or mixed strategies) of the following game

		Player 2	
		Left	*Right*
Player 1	*Top*	6, 2	0, 1
	Bottom	3, 0	6, 4

(20) **Exercise 2:** (Final 2017) Consider the following game between the taxpayer and an auditor. The taxpayer can save x euros by evading taxes. But if a tax-evader is audited, he has to pay a fine f which is higher than the taxes saved x. The auditor incurs cost c when she decides to audit. But if she catches an evasion, she receives a reward r which is higher than the auditing cost c. Assuming that each player is maximizing the expected monetary returns (ignoring ethical considerations), the payoff matrix can be written as follows:

	Audit	*No Audit*
Evade	$x - f, r - c$	$x, 0$
Not Evade	$0, -c$	$0, 0$

Find the Nash equilibria of this game. Government officials want to reduce the rate of tax evasion. What is your recommendation? Should they change the fine level? Should they change the reward paid to the auditor? What should be the direction of the change(s)? Explain briefly.

(25) **Exercise 3:** (Midterm 2017) Consider the following game where the pure strategies for Player 1 are U and D, and the pure strategies for Player 2 are L and R:

	L	R
U	3, 0	1, 0
D	1, 2	3, 1

(5) a) Is there a *strictly dominated* pure strategy for any of the players? If yes, which one?

(5) b) Is there a *weakly dominated* pure strategy for any of the players? If yes, which one?

(5) c) Are there Nash equilibria of this game in *pure strategies*? If yes, find them.

(10) d) Find *all* the Nash equilibria of this game.

(25 points) **Exercise 4:** (Midterm 2018) Consider the game with the following payoff matrix, played

by players 1 and 2:

Player 2

Player 1		*Left*	*Right*
	Top	x, y	$0, 0$
	Bottom	$0, 0$	$1, 1$

where x and y are **positive numbers**. As usual, the first payoff in each cell belongs to player 1 - who chooses between the rows and the second payoff belongs to player 2 - who chooses between the columns.

(5) a) Find the *pure strategy* Nash equilibria of this game.

For the rest of the exercise, consider the mixed strategies.

(10) b) Write down the best response correspondence of each player in this game.

(5) c) Find all the Nash equilibria of this game.

(5) d) Suppose that the players are playing a Nash equilibrium of this game in *completely* mixed strategies (not in pure strategies). How would increasing the payoff x change the probability that player 1 choose to play *Top*? Explain in one or two sentences.

Exercise 5: (Final 2018) Consider the following game between two players. Each of the players will simultaneously choose a number. The player who chooses a (strictly) higher number wins and the other one loses. If they choose the same number, there is a tie between the players. Each player prefers to win rather than to tie; and he prefers to tie, rather than to lose. Is there a Nash equilibrium of this game? Reconcile your answer with the existence theorem of Nash.

Exercise 6: (Midterm 2019, 20 points) Bad news. Alex and Chris got separated. It was a painful breakup and they do not talk to each other anymore. But life goes on. Now each of them has to decide on which classes to register for the Spring term. There are two options. Option A is a course on *Advanced Game Theory*. Option B is *Basics of the Astrological Dimensions of Business*. Since it will be painful to see each other in every lecture, Alex and Chris will both get zero payoff if they register to the same class. If they choose to register to different classes, the payoffs are positive, but the one who chooses the advanced game theory course will get x times as much payoff as the other.

(10) a) Write down the normal form (with a payoff matrix) and find all Nash equilibria (in pure and mixed strategies) of this game.

(10) b) You should notice that one of the equilibria you found above is symmetric, i.e. both players follow the same strategy. Find out if this symmetric equilibrium strategy is evolutionary stable: Assume that this strategy describes the proportions of players in a large population that are programmed to make different choices each time they are randomly matched with another player. Suppose the payoffs give the relative survival rates of the players. If the equilibrium proportions are modified, would the evolutionary forces push them back to the initial state in this game? Why or why not?

Exercise 7: (Final 2019, 30 points) *Nash Equilibrium.* In a tennis match, each time a player serves the ball, she can choose whether to aim the ball to the right or to the left of her opponent. The receiving player must also decide whether to be more prepared for a right-directed ball or for a left-directed ball the very moment the server makes her serve, since the ball travels very fast. The probability that the server wins the point depends on the strategies chosen by the two players. Assuming that each player chooses her strategy to maximize the probability that she wins the point, we can write the payoff matrix for their interaction as below:

		Receiver	
		Right	Left
Server	Right	$0.1, 0.9$	$0.7, 0.3$
	Left	$0.8, 0.2$	$0.4, 0.6$

Find all the Nash equilibria of this game (in pure or mixed strategies). If the players are indeed playing a Nash equilibrium, what is the probability that the server wins the point?

Exercise 8: (Midterm 2020, 25 points) Consider the following case study from 1970s: Polaroid is the market leader for instant photography. Kodak will choose whether to enter *in* the instant photography market or to stay *out*. Polaroid either *accommodates* or *fights* a potential entry. The payoffs are given by the following payoff matrix:

		Polaroid	
		Acc	$Fight$
Kodak	In	$2, 1$	$0, 0$
	Out	$1, 2$	$1, 2$

(5) a) Find the *strictly* and *weakly* dominated pure strategies for both firms.

(10) b) Find all the *Nash equilibria* of the game (in pure and mixed strategies).

(10) c) *Trembling-Hand-Perfect* Nash Equilibrium: Consider a modification of the game where the two players are not allowed to follow pure strategies: Suppose each player is required to choose a mixed strategy where probability of playing either pure strategy has to be at least ε, where ε is a strictly positive but very small number. Find the set of Nash equilibria of this modified game. What happens to this set as ε converges to zero?

Exercise 9: (Final 2020, 15 points) Players 1, 2, and 3 face the risk of catching a new viral disease. The disease will be avoided by all if *at least two* out of the three players follow the rules of "social distancing." Avoiding the disease in this way increases the payoff of each of the three players by 100 units. The individual cost of following social distancing for a player is 18 units of payoff. Consider the simultaneous move game where each of the three players simultaneously decides whether to respect the rules of social distancing or not.

(3) a) Are there any *dominated* (weakly or strictly) strategies in this game? Either tell what these strategies are, or explain briefly why they do not exist.

(5) b) Find all the *pure-strategy Nash equilibria* of this game.

(2) c) What is player 1's expected payoff (as a function of p) if he does not follow social distancing measures?

(3) d) What are the values of p that will make player 1 indifferent between following social distancing or not following it?

(2) e) In light of your answer to the question above, find the *symmetric mixed-strategy equilibria* of this game.