Lecture 2

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January 24th, 2022

Overview

- Common knowledge and the beauty contest
- Iterated Elimination of Strictly Dominated Strategies
- Introduction to Nash Equilibrium

Overview

- **Common knowledge and the beauty contest**
- Iterated Elimination of Strictly Dominated Strategies
- Introduction to Nash Equilibrium

Common knowledge and the beauty contest

Let's play a game. . .

- Each player chooses an integer between 0 and 100.
- We calculate the average of the submitted numbers.
- The aim of each player is to choose an integer which is as close as possible to the 2/3 of this average.
- This game is a version of Keynes's Beauty Contest.

Common knowledge and the beauty contest

Common knowledge and the beauty contest

- John Maynard Keynes (1883-1946), British economist.
- Suppose a newspaper is organizing a beauty contest.
- The readers are asked to vote for the prettiest contestant.
- The contestant who receives the highest support is chosen as Miss Great Britain.
- The readers who voted for her receive prizes.

Common knowledge and the beauty contest

- Who would you vote for?
  - The prettiest?

Common knowledge and the beauty contest

- Who would you vote for?
  - The prettiest?
  - The one you think the others would find prettiest?

Common knowledge and the beauty contest

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Common knowledge and the beauty contest

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Common knowledge and the beauty contest

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- Here is what Keynes says about this:
- "It is not a case of choosing those which are really the prettiest, nor even those which average opinion genuinely thinks prettiest...

Common knowledge and the beauty contest

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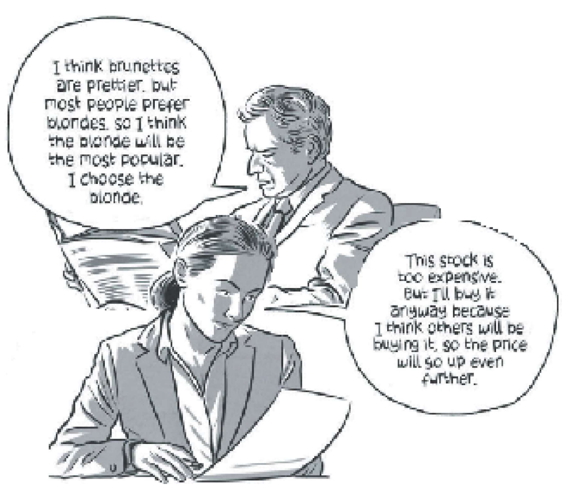
  - The prettiest?
  - The one you think the others would find prettiest?
  - The one you think the others would think the others would find prettiest?
- Here is what Keynes says about this:
- "It is not a case of choosing those which are really the prettiest, nor even those which average opinion genuinely thinks prettiest...
- ...We devote our intelligences to anticipating what average opinion expects the average opinion to be."

Common knowledge and the beauty contest

- Keynes is the father of macroeconomics.
- What does beauty contest game have to do with economics?
- Success in financial markets depends on being one step ahead of the crowd.
- If you can predict the behavior of the average investor, you are the most successful of all.

Common knowledge and the beauty contest

Introducing Game Theory, A Graphic Guide by Ivan Pastine, Tuvana Pastine & Tom Humberstone



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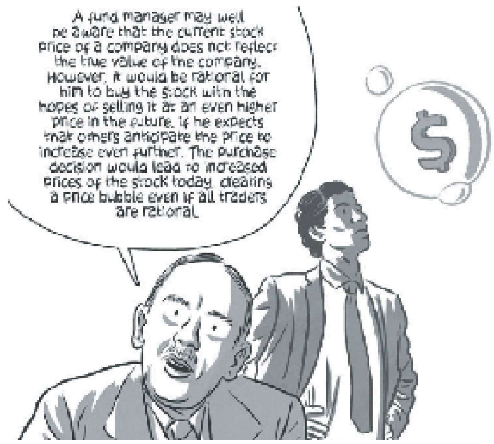

Overview

- Common knowledge and the beauty contest
- **Iterated Elimination of Strictly Dominated Strategies**
- Introduction to Nash Equilibrium

Back to the Prisoners' Dilemma

- Let's imagine a modified version of the original game where Player 1 is the brother of the chief of police.
- If both players deny, Player 1 is set free.
- There is no change in the other prison terms.

v_1 / v_2	deny	confess
deny	$0, -2$	$-5, -1$
confess	$-1, -5$	$-4, -4$

- Will Player 2 ever play "deny"?
- Since $-1 > -2$ and $-4 > -5$, "deny" is a dominated strategy for Player 2.

Back to the Prisoners' Dilemma

- Player 1 knows this and will account for it:

v_1 / v_2	confess
deny	$-5, -1$
confess	$-4, -4$

- Player 2 plays confess $(-4 > -5)$.
- Assumptions:
  - Player 2 is rational (i.e. will not play a dominated strategy)

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  - Player 1 is rational...

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- Assumptions:
  - Player 2 is rational (i.e. will not play a dominated strategy)
  - Player 1 is rational...
  - ...and knows that Player 2 is rational

Back to the Prisoners' Dilemma

- Can we make a prediction about the following?

v_1 / v_2	left	center	right
top	3, 3	1, 2	1, 1
middle	1, 4	2, 3	2, 2
bottom	0, 3	1, 5	4, 4

- "right" is strictly dominated by "center"

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- "right" is strictly dominated by "center"
- In the reduced game, "bottom" is dominated by "middle"

Back to the Prisoners' Dilemma

- Can we make a prediction about the following?

v_1 / v_2	left	center	right
top	3, 3	1, 2	1, 1
middle	1, 4	2, 3	2, 2
bottom	0, 3	1, 5	4, 4

- "right" is strictly dominated by "center"
- In the reduced game, "bottom" is dominated by "middle"
- In the re-reduced game, "left" dominates "center"

Back to the Prisoners' Dilemma

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v_1 / v_2	left	center	right
top	3, 3	1, 2	1, 1
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- "right" is strictly dominated by "center"
- In the reduced game, "bottom" is dominated by "middle"
- In the re-reduced game, "left" dominates "center"
- Eventually "top" dominates "middle"

Back to the Prisoners' Dilemma

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top	3, 3	1, 2	1, 1
middle	1, 4	2, 3	2, 2
bottom	0, 3	1, 5	4, 4

- "right" is strictly dominated by "center"
- In the reduced game, "bottom" is dominated by "middle"
- In the re-reduced game, "left" dominates "center"
- Eventually "top" dominates "middle"
- Our prediction is "top", "left".

Iterated Elimination of Strictly Dominated Strategies

- The name of this process is "Iterated Elimination of Strictly Dominated Strategies"
- Iterated-elimination Equilibrium
- Assumptions to reach it:
  - Player 1 is rational.

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  - 1 knows that 2 knows that 1 is rational and vice versa.

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  - 1 knows that 2 knows that 1 is rational and vice versa.
  - ...

Iterated Elimination of Strictly Dominated Strategies

The formal assumption:

- Common Knowledge of Rationality:
- Player i knows that Player j knows that Player i knows that . . . . Player i is rational.
- We can extend the statement above as long as necessary (infinitely many times).
- Notice that this is stronger than rationality of all players.

Overview

- Common knowledge and the beauty contest
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- **Introduction to Nash Equilibrium**

Best Response

- "What is my best response if the other player chooses x?"

v_1 / v_2	left	center	right
top	3, 3	1, 2	1, 1
middle	1, 4	2, 3	2, 2
bottom	0, 3	1, 5	4, 4

- For P2, best response to "top" is "left", to "middle" is "left", to "bottom" is "center".
- For P1, best response to "left" is "top", to "center" is "middle", to "right" is "bottom".

Best Response (Definition)

Player i's best response function gives her best strategy s_i for a list s_{-i} of other players' strategies:

$$\begin{aligned} b_i(s_{-i}) &= s_i \in S_i \\ & \quad s.t. \\ v_i(s_i, s_{-i}) &\geq v_i(s_i', s_{-i}) \\ \forall s_i' &\in S_i \end{aligned}$$

Best Response (Example)

v_1 / v_2	deny	confess
deny	$-1, -2$	$-5, -1$
confess	$-1, -5$	$-4, -4$

$b_1(\text{deny}) = \{\text{deny,confess}\}$
NB: A player may have more than one best response to the strategies of the others.

Best Response (Example)

v_1 / v_2	left	center	right
top	0, 7	2, 5	7, 0
middle	5, 2	3, 3	5, 2
bottom	7, 0	2, 5	0, 7

- any dominated strategies?
- what about IESDS?
- what about best responses?
- Common knowledge of rationality does not eliminate any of the strategies in this game.
- Any strategy in this game is rationalizable.

Best Response (Example)

v_1 / v_2	left	center	right
top	0, 7	2, 5	7, 0
middle	5, 2	3, 3	5, 2
bottom	7, 0	2, 5	0, 7

- Below is how one can rationalize strategy "top"
  - Player 1 may play "top"
  - thinking that Ply 2 plays "right"
  - thinking that Ply 1 plays "bottom"
  - thinking that Ply 2 plays "left"
  - thinking that Ply 1 plays "top"

Nash Equilibrium

- A strategy profile $s^* = (s_1^*, s_2^*, ..., s_n^*)$ is a **Nash Equilibrium** if

$$v_i\left(s_i^*, s_{-i}^*\right) \geq v_i\left(s_i', s_{-i}^*\right)$$

- In other words, $s^*$is a **Nash Equilibrium** if all players use their best response functions,

$$s_i^* = b_i\left(s_{-i}^*\right), \forall i$$

Never Best response

- A strategy s_i is never best response if $\nexists\ s_{-i}$ s.t. $s_i = b_i(s_{-i})$.
- **Proposition:** A strictly dominated strategy is never best response.
- Is it possible for a strategy that is not strictly dominated to be a never best response?

v_1 / v_2	left	right
A	10	0
B	0	10
C	3	3

- C is not strictly dominated but is Never Best Response.
- C is strictly dominated if Player 1 chooses A or B by flipping a fair coin (mixed strategies).

Never Best response

- **Proposition:** When mixed strategies are taken into account, in 2-player games, if a strategy is never best response then it is strictly dominated.
- formal proof in the textbook by Fudenberg and Tirole

Exercise you can try

- Consider the strategic form game represented by the following bimatrix, where player 1 is the row and player 2 is the column player:

v_1 / v_2	L	R
T	a, b	$c, 2$
M	$1, 1$	$1, 0$
B	$3, 2$	$0, 1$

- a, b and c are numbers which are left unspecified for now.
  - For which values of a, b and c the outcome (T;L) is a strictly dominant strategy equilibrium?

Exercise you can try

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- a, b and c are numbers which are left unspecified for now.
  - For which values of a, b and c the outcome (T;L) is a strictly dominant strategy equilibrium?
  - For which values of a, b and c the outcome (T;L) is a pure-strategy Nash equilibrium.

Exercise you can try

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v_1 / v_2	L	R
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M	$1, 1$	$1, 0$
B	$3, 2$	$0, 1$

- For which values of a, b and c the outcome (T;L) is a strictly dominant strategy equilibrium?
- It must be: $a > \max\{1, 3\} = 3$, $c > \max\{1, 0\} = 1$ and $b > 2$
- For which values of a, b and c the outcome (T;L) is a pure-strategy Nash equilibrium.
- It must be: $a \geq \max\{1, 3\} = 3$ and $b \geq 2$. The value of c is not relevant here.
- What if we want (T;L) to be the only pure-strategy Nash equilibrium?

Exercise you can try

- Consider the strategic form game represented by the following bimatrix, where player 1 is the row and player 2 is the column player:

v_1 / v_2	L	C	R
T	$4, 2$	$2, 6$	$2, 4$
M	$6, 2$	$2, 0$	$2, 4$
B	$2, 10$	$-2, 8$	$0, 4$

- Are there strictly dominated strategies? If yes, which one(s)?
- Find all strategies that survive IESDS.
- Find the Nash Equilibria

Exercise you can try

v_1 / v_2	L	C	R
T	$4, 2$	$2, 6$	$2, 4$
M	$6, 2$	$2, 0$	$2, 4$
B	$2, 10$	$-2, 8$	$0, 4$

- **Strict dominance:** P1: Strategy B is strictly dominated by T $(4 > 2, 2 > -2, 2 > 0)$
- **IESDS:** If we delete B we are left with a 2*3 matrix. In that case, P2 never plays L.
- This leads us to a 2*2 matrix with strategies T, M for P1 and C, R for P2.
- **NE:** In the 2*2 matrix we can see that (T,C) and (M,R) are the Nash equilibria. Can we see the same in the 3*3 matrix? (see slide 24)

Comparison

Up to now we have seen the following:

- Strictly dominant strategy equilibrium (focus on the dominant strategies)
- Iterated elimination equilibrium (focus on the dominant strategies, update the game and repeat)
- N.E. (focus on best responses)

Summary

- Iterated Elimination of Strictly Dominated Strategies
- Best response
- Nash equilibrium

The end

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(do not use the essec email in moodle)