Lecture 1

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Overview

- Game Theory: Motivation and Context
- Static Games of Complete Information
- Prisoner's Dilemma
- Dominance in Pure Strategies
- Other Examples
- Summary

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Strategic vs. Non-strategic Problems

Non-strategic Problems

- Consumer's Problem
- A monopoly owning a market (e.g. YouTube)
- An estimation problem in statistics
- etc

Strategic vs. Non-strategic Problems

Strategic Problems

- Duopoly (e.g. Apple vs. Microsoft)
- Chess
- Rock-paper-scissors
- Nim game

What is the appropriate solution concept?

Strategic vs. Non-strategic Problems

- Strategic interaction: the outcome for a single player (agent) depends on the decisions made by other players (agents).
- Modern game theory has a very large scope.
- Deals with all real-life situation where rational people interact with each other.
- Examples in political science, economics, computer science, international relations, etc.

Game theory

- **Game theory** in the form known to economists, social scientists, and biologists, was given its first general mathematical formulation by John von Neumann and Oskar Morgenstern.
- **John von Neumann** (1903 -1957) Hungarian-American mathematician, University of Budapest and Princeton.
- **Oskar Morgenstern** (January 24, 1902 - July 26, 1977) was an Austrian-American economist, U. Vienna and Princeton
- "Theory of games and economic behavior" (1944).

Game theory

They were followed by many...

- **John Nash**, Princeton, published "Non-cooperative games" (1951). Annals of mathematics, 286-295.
  - Shows existence of equilibrium for non zero sum games.
- **Shizuo Kakutani**, Japanese-American mathematician, Princeton and Yale. Published Kakutani, S. . "A generalization of Brouwer's fixed point theorem" (1941). Duke mathematical journal, 8(3), 457-459.
  - Shows his theorem, and then shows von Neumann and Morgerstern main results in a couple of lines.
- **David Blackwell**, "Equivalent comparisons of experiments" (1953). The annals of mathematical statistics, 265-272.

Game theory

The theory of games of incomplete information took over by the end of the 60's and is in permanent development up until today.

- **Robert Aumann** and **Michael Maschler**, "Repeated Games of Incomplete Information: a Survey of Recent Results" (1967) in Report of the U.S. Arms Control and Disarmament Agency ST-116, Washington, D.C., Chapter III, pp.287-403.
- **John Harsanyi**, (1967) Hungarian-American economist. Games with incomplete information played by "Bayesian" players, I-III. Management science, 14(3), 159-182.
- **John Harsanyi** and **Reinhard Selten**, (1988). A general theory of equilibrium selection in games. MIT Press Books, 1.

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Game theory

All games have three elements:

- **Players:** Each decision-maker in a game is called a player (an individual, a firm, an entire nation.)
- **Strategies**: Each player has the ability to choose among a set of possible strategies (!!!).
- **Payoffs:** The final returns to the players at the end of the game. Depend on all actions chosen.

Definitions (Strategy)

- Each **course** of action open to a player is called a strategy.
- A complete **contingent plan**.
- Strategies can be very simple (rock-paper-scissors) or very complex (nim game).

Definitions (Payoffs)

- The final returns to the players at the end of the game are called payoffs.
- Payoff of a player depends on his strategy as well as the strategies of the other players.
- Payoffs are usually measured in terms of utility (monetary payoffs are also used).
- We assume that players can rank the payoffs associated with a game and take expectations over them (relevant for mixed strategies).
- Players are rational and benefit by common knowledge (unless otherwise specified).

Definitions (Normal Form of a Game)

- A set of players $I = \{1, 2, ..., n\}$
- A collection of strategies $S = \{S_1, S_2, ..., S_n\}$
- A collection of payoff functions $\{v_1, v_2, ..., v_n\}$

Simultaneous move games: For now, we assume that players choose their strategies simultaneously.

Matrix Representation of a game

For two players games, we have:

- Rows. Each row represents one of player's 1 strategies.
- Columns. Each column represents one of player's 2 strategies.
- Matrix entries. Each entry contains two elements $(v_1(\cdot,\cdot), v_2(\cdot,\cdot))$

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Prisoner's Dilemma

The most famous game of all times...

- Two criminals named 1 and 2 have been apprehended by the police and are being questioned separately.
- Each one can choose either to confess or deny.
- If both confess, each gets a 4 year jail sentence.
- If both deny, each gets a 2 year sentence.
- If one confesses and the other denies, the confessor serves 1 year and the denier serves 5 years in jail.

Prisoner's Dilemma

$I = \{1, 2\}, S = \{S_1, S_2\}$

v_1 / v_2	deny	confess
deny	$-2, -2$	$-5, -1$
confess	$-1, -5$	$-4, -4$

- LHS payoff for 1 and RHS for 2.
- Assume that each criminal is trying to minimize his jail term.
- First, Row player (Player 1) has a strict advantage to confessing, no matter what Column player (Player 2) is doing.
- The same argument applies to Column player too (symmetric game).
- Therefore, our prediction for this game is that both players will confess.

Prisoner's Dilemma

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confess	$\underline{-1}, -5$	$\boxed{\underline{-4}, \underline{-4}}$

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Definition: Strict Domination

- For player i, strategy s_i'' **strictly dominates** s_i' if

$$v_i\left(s_i'', s_{-i}\right) > v_i\left(s_i', s_{-i}\right), \forall s_{-i}$$

- s_i' is **strictly dominated** by s_i''
- s_i'' is **strictly dominant** if it strictly dominates all the other strategies of player i.
- In Prisoner's dilemma, "Confess" **strictly dominates** "Deny"
- A strategy profile $s^D = \left(s_1^D, s_2^D, ...s_n^D\right)$ is a **strictly dominant strategy equilibrium** if $s_i^D, \forall i$ is a strictly dominant strategy.
- This is our prediction.

Definition: Weak Domination

- For player i, strategy s_i'' **weakly dominates** s_i' if

$$v_i\left(s_i'', s_{-i}\right) \geq v_i\left(s_i', s_{-i}\right), \forall s_{-i}$$

- s_i' is **weakly dominated** by s_i''
- s_i'' is **weakly dominant** if it weakly dominates all the other strategies of player i.

v_1 / v_2	deny	confess
deny	$\underline{-1}, -2$	$-5, \underline{-1}$
confess	$\underline{-1}, -5$	$\boxed{\underline{-4}, \underline{-4}}$

- For player 1, confess weakly dominates deny since $-4 > -5$ but $-1 = -1$
- For player 2, confess strictly dominates deny since $-4 > -5$ and $-1 > -2$.

Definition: Pareto Efficiency

- Vilfredo Pareto (1848-1923): contributions in microeconomics
- "An outcome is efficient for the society if there is no other alternative outcome which makes someone better off without hurting someone else."
- For instance, if both players play deny then they are both better off.

v_1 / v_2	deny	confess
deny	$-2, -2$	$-5, \underline{-1}$
confess	$\underline{-1}, -5$	$\underline{-4}, \underline{-4}$

- As long as the players play individually, the Pareto efficient outcome is unstable.
- NB: Pareto efficiency is an interesting concept.

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Prisoner's Dilemma: Example

Fits to a wide variety of real-life strategic interactions:

- You are sharing a dormitory room.
- You prefer a tidy room to a messy one.
- You would rather prefer your roommate to clean up the room instead
- of you.
- And you would absolutely not like to clean up your roommate's mess.

v_1 / v_2	Clean	Mess
Clean	2, 2	0, 3
Mess	3, 0	1, 1

Prisoner's Dilemma: Example

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v_1 / v_2	Clean	Mess
Clean	2, 2	$0,\underline{3}$
Mess	$\underline{3},0$	$\boxed{\underline{1},\underline{1}}$

Mafia modified Prisoner's Dilemma

Consider the following payoffs:

v_1 / v_2	deny	confess
deny	$-2, -2$	$-5, -1-z$
confess	$-1-z, -5$	$-4-z, -4-z$

If $z > 1$ (e.g. $z = 2$) we have:

v_1 / v_2	deny	confess
deny	$\boxed{\underline{-2}, \underline{-2}}$	$\underline{-5}, -3$
confess	$-3, \underline{-5}$	$-7, -7$

Tragedy of the commons (Garett Hardin)

- Prisoners' dilemma is not necessarily a two-player phenomenon.
- A piece of land that villagers let their cows graze.
- Each villager decides how many cows to keep.
- The villager receives the benefits from an additional cow.
- But the damage to the common is shared by the entire group.
- Fisheries, Water resources, Global warming, Arms race

Ways to achieve efficiency

- Contracts / Commitments? (may be illegal/infeasible)
- Repeated Interactions (criminals, roommates)
- Reputation

Exercises you can try

v_1 / v_2	a	b
a	$0, 0$	$2, -1$
b	$-1, 2$	$1, 1$

v_1 / v_2	a	b
a	$0, 0$	$2, -2$
b	$-1, -1$	$1, 1$

Study the games (players, strategies, payoffs, dominance, Pareto efficiency).

Exercises you can try

v_1 / v_2	R	P	S
R	$0, 0$	$-1, 1$	$1, -1$
P	$1, -1$	$0, 0$	$-1, 1$
S	$-1, 1$	$1, -1$	$0, 0$

v_1 / v_2	L	M	H
L	$6, 6$	$2, 8$	$0, 4$
M	$8, 2$	$4, 4$	$1, 3$
H	$4, 0$	$3, 1$	$2, 2$

Study the games (players, strategies, payoffs, dominance, Pareto efficiency).

Summary

- Context and development of Game Theory.
- Components of a game
- strict/weak dominance
- Pareto Optimality.
- Examples

The end

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(do not use the essec email in moodle)