



# GAME THEORY

## Assignment 3

### Instructions

You must answer all parts in both exercises of this assignment. Each exercise carries equal weight. The weights of sub-questions are given in parentheses. This is an individual project and accounts for 10% of your final grade on this course. The project is due by 21.00 on 11/06/2024. You can submit a handwritten copy in class, or you can send it via email to [efilipp@uom.edu.gr](mailto:efilipp@uom.edu.gr). Should you choose to send your assignment via email you must submit a SINGLE FILE (either scan your handwritten copy or submit a typed assignment). Late assignments WON'T be accepted.

**Exercise 1 (50%)** Consider a static Bayesian game with two players where Nature decides whether player 2 is type A with probability  $p$  or type B with probability  $(1 - p)$ . The corresponding payoffs are as in payoff matrices A and B (see below). Player 2 is informed about her “type” while player 1 only knows the probability distribution over the types.

**Payoff Matrix A**

		Player 2 <sup>A</sup>	
		Left	Right
Player 1	Up	1, 0	0, 2
	Down	0, 1	2, 0

**Payoff Matrix B**

		Player 2 <sup>B</sup>	
		Left	Right
Player 1	Up	1, 1	0, 0
	Down	0, 0	2, 2

- What are the strategy sets of the two players? (*remember: player 2 has two types!*) (7 marks)
- Construct the unified payoff matrix and fill the respective payoffs. (18 marks)
- Properly report all Bayesian Nash Equilibria (BNE) of this game. (18 marks)
- Under what conditions does this game have (i) a unique BNE, and (ii) no BNE? (7 marks)

**Exercise 2 (50%)** Consider a First-Price Sealed Bid (FPSB) auction with independent private values where a single prize is to be awarded to one out of  $n$  bidders. According to the rules of the FPSB auction:

- The highest bidder wins the object. If there are two or more bids tied at the top, the winner is chosen randomly (where all top bidders win with positive probability).
- Only the winning bidder pays. The price paid for the object is denoted by  $p$ .
- Bidders are risk-neutral. If  $i$  wins the object and the price is  $p$ , his utility is  $v_i - p$ . All non-winners have a utility of 0.

Independent valuations,  $v_i$  for each  $i$ , are drawn from a continuous uniform probability distribution.

Find the Bayesian Nash Equilibrium (BNE) of this incomplete information game.