# GAME THEORY

Assignment 2

Instructions

You must answer all questions of this assignment. Each question carries equal weight. This is an individual project and accounts for 10% of your final grade on this course. The project is due by 21.00 on 03/06/2024. You can submit a handwritten copy in class, or you can send it via email to efilipp@uom.edu.gr. Should you choose to send your assignment via email you must submit a **SINGLE FILE** (either scan your handwritten copy or submit a typed assignment). Late assignments WON'T be accepted.

Note: also available for you to solve is the second question of the first assignment (in case you haven't solved that). This will be awarded 0,40 marks of the final grade.

Consider an extensive form game between two players as described by the game tree below:
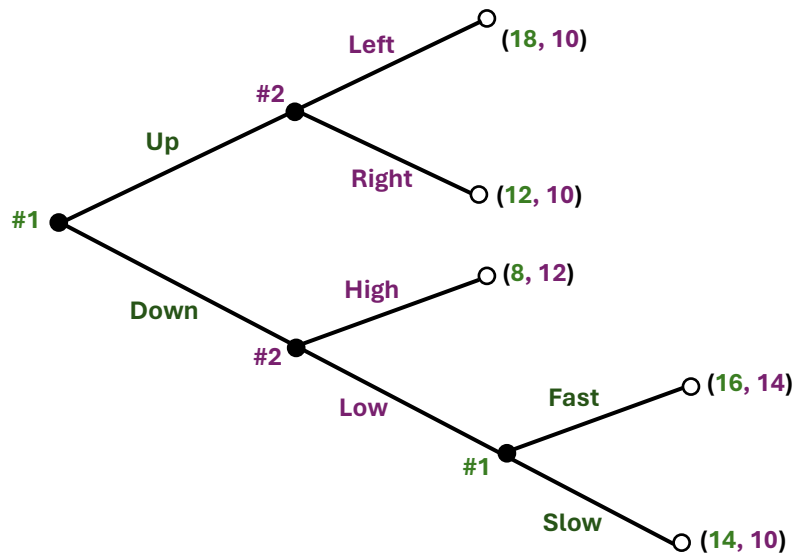
a. What are the strategy sets for the two players? (10 marks) *(Remember: a strategy must include that many actions as the number of decision nodes)*

b. Properly describe/present all the subgames of this game. (10 marks) *(You can clearly indicate the subgames on the game tree and/or you can verbally describe them)*

c. Find all Subgame Perfect Nash Equilibria and properly describe them. What is the backwards induction outcome that corresponds to each of the SPNEs?

Exercise 2 (50%) Two different women, Maria and Elena, claim to be the owners of an antique painting of sentimental value. Let C_M denote Maria's and C_E Elena's monetary equivalent of getting the painting. Not getting the painting yields, for both, a monetary equivalent of zero euros. However, none of them possesses evidence of ownership. A judge wants the painting to be returned to its legal owner. He knows that one of the two is indeed the legal owner, but he does not know who that is. He comes with the following idea. A fine $F > 0$ is announced and then the judge asks Maria and Elena to play the following game.

- In the first stage, Maria either gives up her claim to the painting (in which case the game ends with Elena getting the painting and nobody paying the fine) or she asserts her claim, in which case the game proceeds.

- In the second stage, Elena either accepts Maria's claim (in which case the game ends with Maria getting the painting and nobody paying the fine) or challenges her claim. In the latter case, Elena must put in a bid $B > 0$ and the game proceeds.

- In the final stage, Maria can either match Elena's bid (in which case Maria gets the painting by paying $F + B$, and Elena pays the fine F) or chooses not to match (in which case Elena pays B and gets the painting while Maria pays F).

Under what conditions a sub-game perfect Nash equilibrium with the painting returning to its legal owner can be supported?

Hints: (1) try to draw a game tree! (2) Work on two different assumptions: what is Maria is the legal owner? What if Elena is?

Assignment 1 - Exercise 2 (0,4 of the final grade) Consider a scenario involving 5 players. Each player i is required to simultaneously submit an integer, x_i, between 1 and 100 alongside the other players. The player whose submitted number is closest to half the average of all submitted numbers, i.e. closer to

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{10}$$

will be awarded a prize (say, 3000 euros; the value of the prize is irrelevant to the problem). Should there be multiple winners, the prize will be divided equally among them. Those who do not win will receive nothing. Determine the Nash Equilibrium of this game.