
# Lecture 3

## Social choice theory

# Social choice theory

- As we have already discussed in the previous lectures, in cases where markets fail to achieve the socially optimal outcome, government intervention is imperative.

- The economic choices made by the government are examined by the social choice theory.

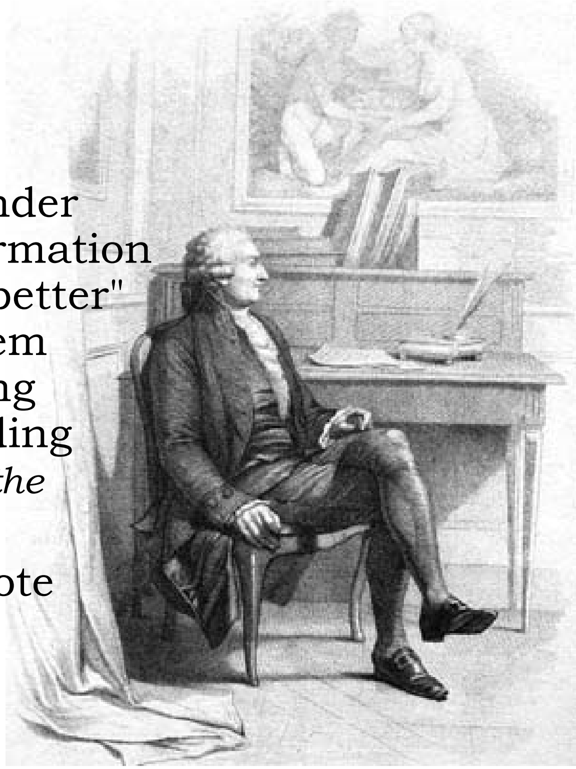
- The central concepts of the theory of social choices relate to the criteria by which social welfare is assessed, which are:
  - economic efficiency,
  - equality and
  - fairness.

# Social choice theory

- The key question is how we can make collective decisions, as these decisions have consequences that, in most cases, do not have the same effect on all individuals but leave some winners and some losers.
  - The Pareto efficiency criterion, used heavily in economics, provides answers only when no one is harmed, and that is why is not very useful in practice.

- In the effort to determine collective preferences, many questions arise, the most important of which are:
  - How can a group of individuals choose among the many alternatives the prevailing one? (e.g., a policy, a candidate, etc)
  - What are the properties of different electoral systems?
  - When is a voting system democratic?
  - How can a college (e.g., electoral, legislative, panel of experts, etc.) identify collective preferences or collective choices in a subject based on individual preferences or individual choices?
  - How can we prioritize different social choices in a list according to social benefit? (Criteria: effectiveness, equality and justice)

# Social choice theory



- *Condorcet's jury theorem*: The theorem establishes that under certain conditions a majority of a group, with limited information about a pair of alternatives, is more likely to choose the "better" alternative than any one member of the group. The theorem thus provides a mathematical basis for majority-rule voting and potentially gives an important clue to our understanding of the strength of democratic government. (1785) *Essay on the Application of Analysis to the Probability of Majority Decisions*.
- However, the conditions are very restrictive: individuals vote independently and they share a common goal.

- *Condorcet's paradox,* states that collective preferences can be cyclic (not transitive), even if the preferences of individual voters are not cyclic.
- *Arrow's impossibility theorem*: states that a clear order of preferences cannot be determined while adhering to mandatory principles of fair voting procedures (see next slide).

# Social choice theory

- Arrow's (impossibility) Theorem (AT): K. Arrow (1951) proved that any social choice rule that satisfies a basic set of fairness conditions could produce illogical results. The conditions are:
  1. Individuals may have any transitive preferences (axiom of unrestricted domain).
  2. If alternative1 is unanimously preferred by all individuals over alternative 2, then alternative 2 should not be chosen (axiom of Pareto choice).
  3. The ranking of two alternatives should not depend on what other alternatives are available (axiom of independence).
  4. No one person should have dictatorial power (axiom of non-dictatorship).

Example of transitivity property failure:

2nd 1st

| | Project X | Project Y | Project Z |
|---|---|---|---|
| Voter 1 | 1 | 2 | 3 |
| Voter 2 | 3 | 1 | 2 |
| Voter 3 | 2 | 3 | 1 |

The order of voting affects the outcome

Result: 2 vote for X and 1 votes for Y

Result: 2 vote for Y and 1 votes for Z

Although the transitivity property holds for individuals' preferences, this property does not transfer to collective decision making. It is clear that although Y is preferred over Z and X over Y, the transitivity property will dictate that X should be preferred over Z. However, in our example, Z is preferred over X. Therefore, aggregation of preferences violates the transitivity property.

## Social choice theory

- Arrow's impossibility theorem, apart from initiating a long discussion around social welfare and giving birth to social choice theory, demonstrated that there is no objective, unambiguous way of defining a rule to guide social choices.

- Thus crucial questions remain open including:
  - How do you add profits and losses occurring to different individuals in a society?
  - How can a society transition from individual to collective preferences?

- In what follows we will consider some the problems associated with Arrow's result, by examining some voting examples first and then moving to the construction of a social welfare function and discuss the problems associated with this. Finally, given that societies need to make choices, some criterion is needed to provide guidance and we will examine the criteria used in economic theory, namely the strict Pareto criterion and the Caldor Hicks criterion.

# Social choice theory

- Example: voting systems with 3 candidates (X, Y, Z)

- **Plurality rule** (one vote to one candidate)

| 40% | 35% | 25% |
| --- | --- | --- |
| X | Y | Z |

  - X has a majority and wins the elections

The two voting rules result in different outcome

- **Majority rule** (ranking of three candidates)

| 40% | 35% | 25% |
| --- | --- | --- |
| X | Y | Z |
| Y | Z | Y |
| Z | X | X |

- 35%+25%=60% $Y \succ X$ and 40%+35%=75% $Y \succ Z$. Therefore, the majority prefers Y to both X and Z

# Social choice theory

- Therefore, since different rules yield different social choices, which voting rule are we going to choose?

- Arrow defined the following selection criteria (i.e. properties that the voting rule should adhere to):
- Decisiveness (there will always be a winner and there can be no more than one winner)
- Pareto principle (if everyone prefers X to Y then X is elected)
- Non-dictatorship (one alone does not impose his choice)
- Independence of irrelevant alternatives (If X is preferred to both Z and Y by a majority, with Z getting a very small percentage (so he has no chance of being elected), then X should also be selected if Z is not a candidate.

Example:

$Y \succ Z$ (68% to 32%) & $X \succ Y$ (67% to 33%)

One would expect that $X \succ Z$, but the result is: $Z \succ X$ (65% to 35%)

Cordorcet paradox

| 35% | 33% | 32% |
|---|---|---|
| X | Y | Z |
| Y | Z | X |
| Z | X | Y |

Violation of the decisiveness criterion

That is, the votes Z gets should not determine the result (i.e. the following case should be excluded: Z receiving votes that if she was not a candidate would go in their majority to Y and give him a majority, overturning the result. Example, 2000 USA elections, Bush, Gore, Nader).

# Social choice theory

- Let us examine a different voting rule and an example of a violation of the criterion of independence from unrelated alternatives. The rule is the Rank-order point (Borda count prefer aggregation rule) according to which each voter assigns a score to each alternative; the most preferred alternative gets a *k* rating (where *k* = number of alternatives), the second most-preferred alternative a score of $k - 1$, etc. The alternative with the largest total sum-is at the top , the alternative with the second largest total sum next, and so on).

| | 1 | 2-7 | 8-15 |
|---|---|---|---|
| 1st | Y | X | Z |
| 2nd | X | Z | X |
| 3nd | Z | W | Y |
| 4th | W | Y | W |

Borda scores:

*x*: 9*3 + 6*4 = 51,
*y*: 1*4 + 6*1 + 8*2 = 26,
*z*: 1*2 + 6*3 + 8*4 = 52,
*w*: 1*1 + 6*2 + 8*1 = 21.

Social preferences: *z* to *x* *and x to y* and *y* to *w.*

15 voters with preferences for four alternatives.
There are two groups (6 and 8 people) that have different preferences and there is one individual with completely different preferences from both these groups

The criterion of independence of irrelevant alternatives does not apply

| | 1 | 2-7 | 8-15 |
|---|---|---|---|
| 1st | X | X | Z |
| 2nd | Y | Z | X |
| 3nd | W | W | Y |
| 4th | Z | Y | W |

Borda scores:

*x*: 7*4 + 8*3 = 52,
*y*: 1*3 + 6*1 + 8*2 = 25,
*z*: 1*1 + 6*3 + 8*4 = 51,
*w*: 7*2 + 8*1 = 22.

Social preferences: *x* to *z* and *z* to *y* and *y* to *w*.

# Social choice theory

- A more general approach to social well-being can be seen by looking at what economists call a social welfare function.

- This function takes into account the levels of well-being of individuals in order to give an indicator of social well-being.

- Although the social welfare function is not a tool that we can use to make social policy decisions, in some cases it is a very useful tool that is widely used.

- The social welfare function takes the general form:

$$W = W(U_1, U_2, \ldots, U_n)$$

where $U_i(X_1, \ldots, X_m)$ denotes the utility function of the individual $i$, $i=1,2,\ldots n$, which depends on her consumption of $m$ good and services. The above function reflects the assumption that social well-being is a function of the well-being of individuals in the society whose objective is to maximize utility from consuming goods and services (we will extent it to include environment).

# Social choice theory

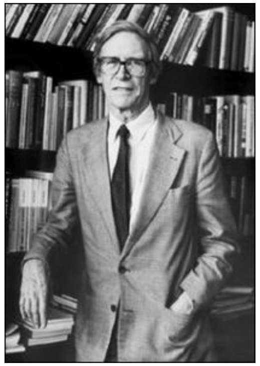
- There are different forms the social welfare function can take, reflecting different societal views and objectives:

  - The Benthamite or utilitarian social welfare function presents society's welfare as the weighted sum of its members' utilities

$$W(U_1, U_2, \ldots, U_n) = \sum_{i=1}^{n} w_i U_i$$

  - where $w_i \geq 0$ are the weights representing the degree of substitutability among individuals' utilities.

  - At one extreme, the weights could be equal. At the other extreme, the Rawlsian or max–min social welfare function presents society's welfare as the utility of its least endowed member,



$$W(U_1, U_2, \ldots, U_n) = \min(U_1, U_2, \ldots, U_n)$$

  - In between the unweighted utilitarian social welfare function and a complete aversion to uncertainty, as expressed by the Rawlsian social welfare function, there is a variety of different approaches.

# Social choice theory

- It is clear that any policy (be it environmental, competition, fiscal, etc) or any public project (such as construction of roads, schools, hospitals, etc) will lead to some type of redistribution of resources that will change social welfare

$$dW = \sum_{i=1}^{n}\sum_{j=1}^{m} \boxed{\frac{\partial W}{\partial U_i}} \boxed{\frac{\partial U_i}{\partial X_{ij}} dX_{ij}}$$

όπου $X_{ij}$ είναι η ποσότητα του αγαθού $j$ που καταναλώνει το άτομο $i$.

- The effect of this redistribution of resources on social welfare could, in general, be split into two parts:

$$dW = \boxed{\sum_{i=1}^{n}\sum_{j=1}^{m} P_j dX_{ij}} + \boxed{\sum_{i=1}^{n}\sum_{j=1}^{m} (\gamma_i - 1) P_j dX_{ij}}$$

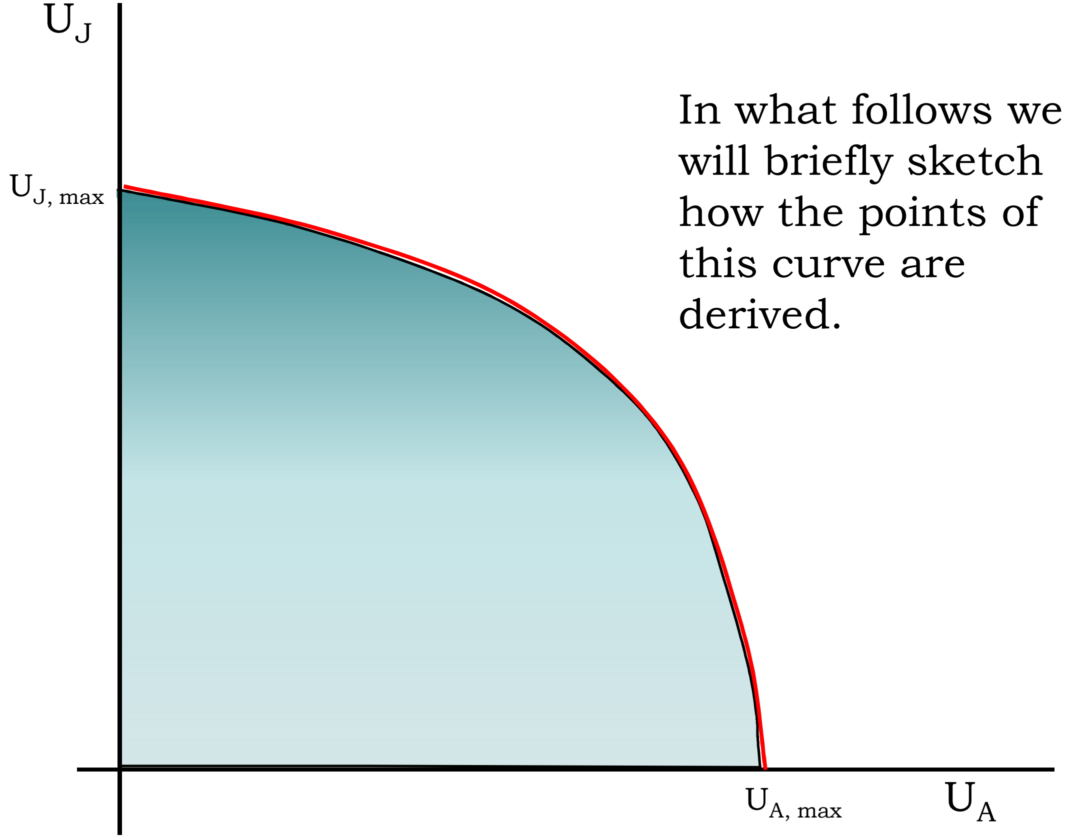
$\gamma_i=(\partial W/\partial U_i)(\partial U_i/\partial Y_i)$ is the marginal social utility of income for individual $i$.

- A part that reflects the effect on efficiency (measured by the change in the value of goods and services) and a part that reflects the evaluation by the society of the change in the distribution of goods and services among the individual members of the society.
  - Assuming that all members, regardless of their status –poor, rich, disadvantaged, etc– have the same weight for the society, then the only effect that matters is the efficiency (i.e. a policy/project that increases rich peoples welfare by €500 while decreasing the poor individuals' welfare by €499 is accepted).

# Utility possibilities frontier

- Society strives to achieve efficiency in both production and exchange. We will discuss efficiency using a diagrammatic analysis that will lead us to the two basic theorems in welfare economics.
- Let us assume a society of two members only Artemis and John *(A, J)* possessing given amounts of two productive resources, *K* and *L*, and receiving value from the consumption of two goods 1 & 2.

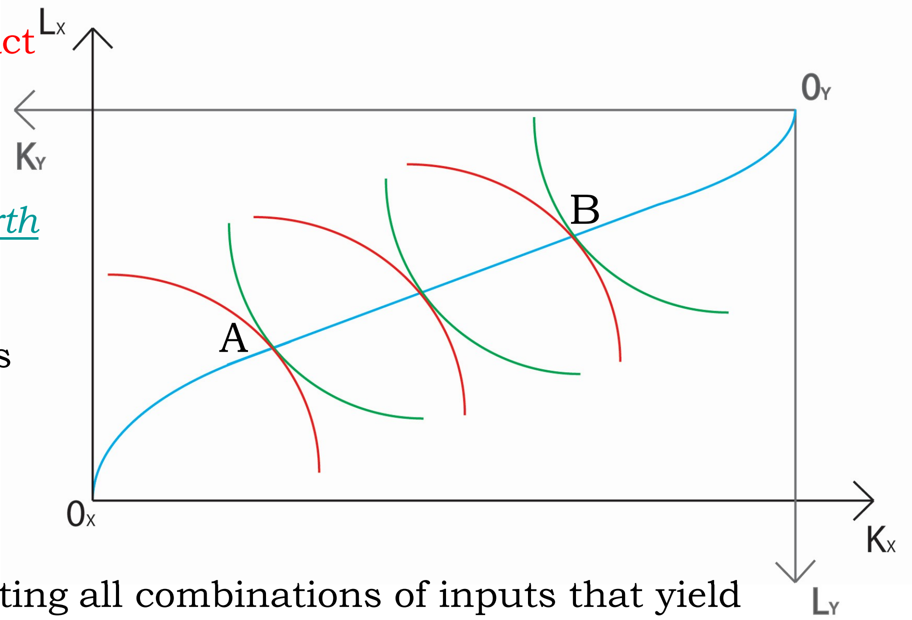
The curve $U_{A,max}U_{J,max}$ illustrates the society's abilities to derive utility, i.e. the maximum (effective) combinations of utility that this society can achieve given the quantities of goods and services that produces (using the resources available and the technology).



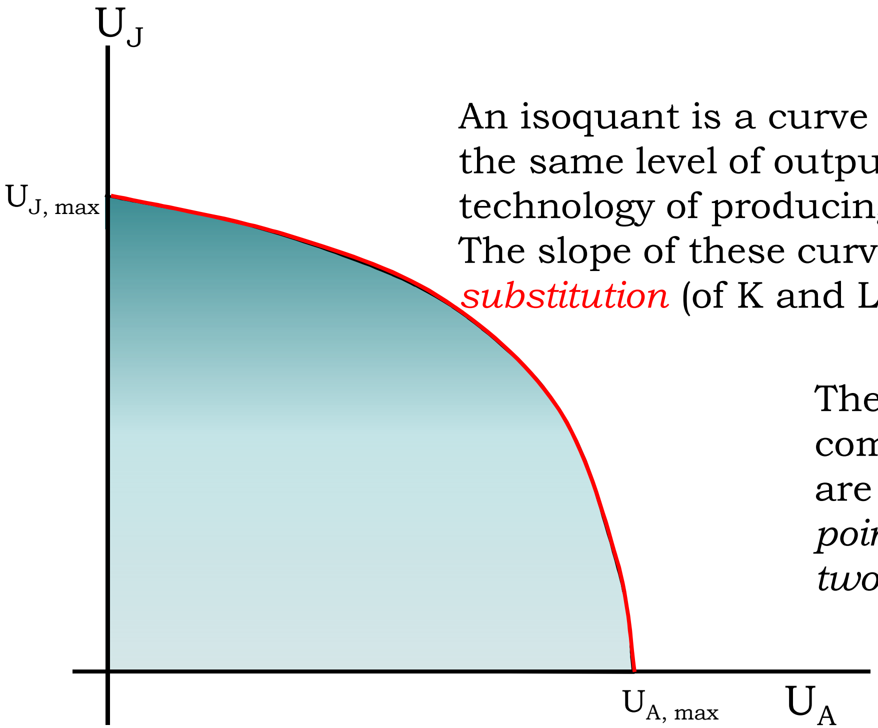

In what follows we will briefly sketch how the points of this curve are derived.

# Contract curve on inputs

We start with what we call in economics the Contract curve along which we achieve technological efficiency.




The graph. on the upper right corner is an *Edgeworth box the dimensions of which are defined by the amounts of the two* inputs (K, L) available to the society of Artemis and John. Different combinations of the two inputs produce different sets of the two goods 1 and 2 call them (X,Y).




An isoquant is a curve presenting all combinations of inputs that yield the same level of output. In the graph, green isoquants illustrate technology of producing X and red isoquants technology of producing Y. The slope of these curves is given by the *marginal rate of technical substitution* (of K and L) for each output X and Y.

The points where the isoquants of different outputs combination intersect (such as points A and B), which are *Pareto-optimal (that means that moving from these points would reduce the production of at least one of the two goods)*, allow us to draw the contract curve.

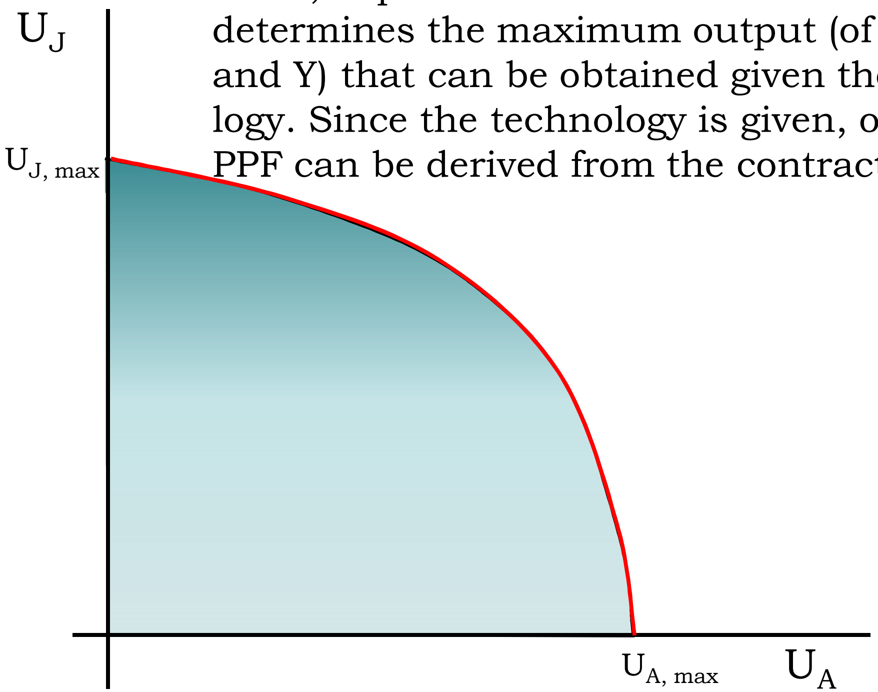
$$MRTS^1_{K,L} = \frac{dL}{dK} = MRTS^2_{K,L}$$

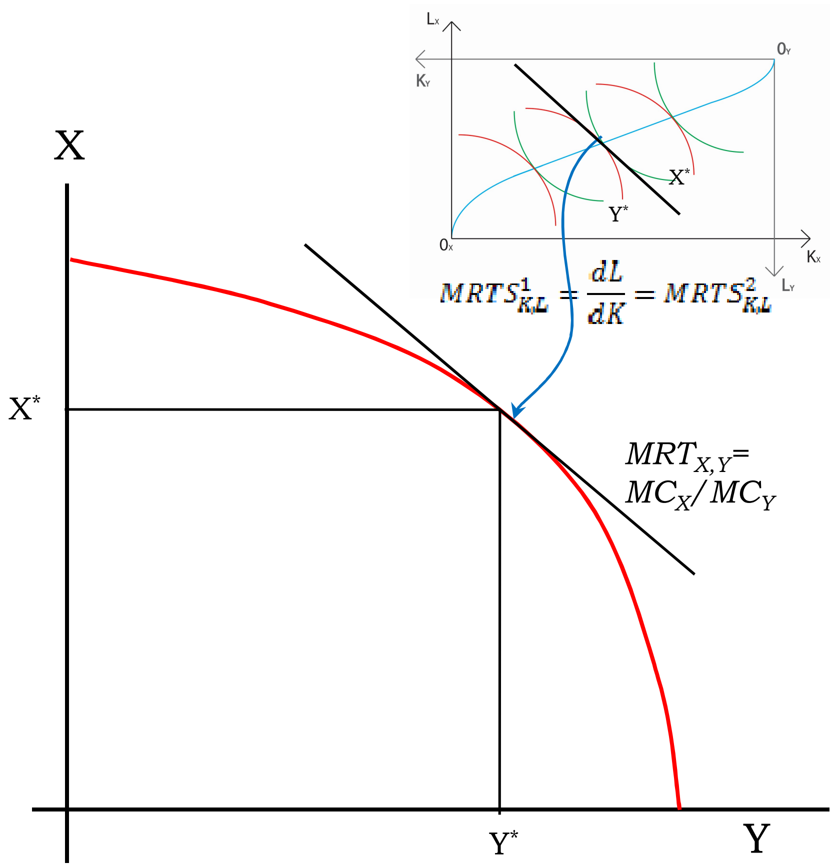
# Production possibilities frontier

And we move to define the Production possibilities frontier (PPF) (on the space of the two goods X and Y) along which technological efficiency is achieved

Lets take an efficient set of outputs produced along the contract curve, let say X* and Y*. The *marginal rate of technical substitution* for X and Y are equal.

Mapping all points on the contract curve to the X,Y space we draw the PPF. This frontier determines the maximum output (of both X and Y) that can be obtained given the techno-logy. Since the technology is given, only one PPF can be derived from the contract curve



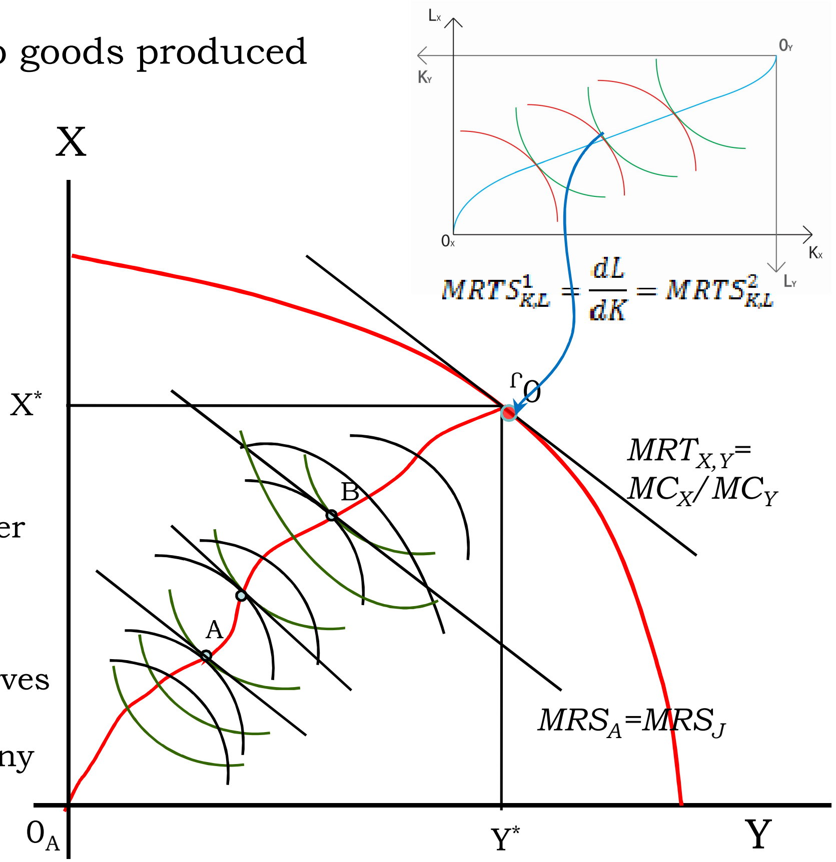




The PPF (red curve) slopes downwards. This slope, which equals the *marginal rate of transformation* (MRT) between X and Y, shows us how, in order to increase the output X, the quantity of Y must decrease. In fact, the marginal rate of transformation measures the tradeoff of producing more X in terms of Y.

# Exchange contract curve

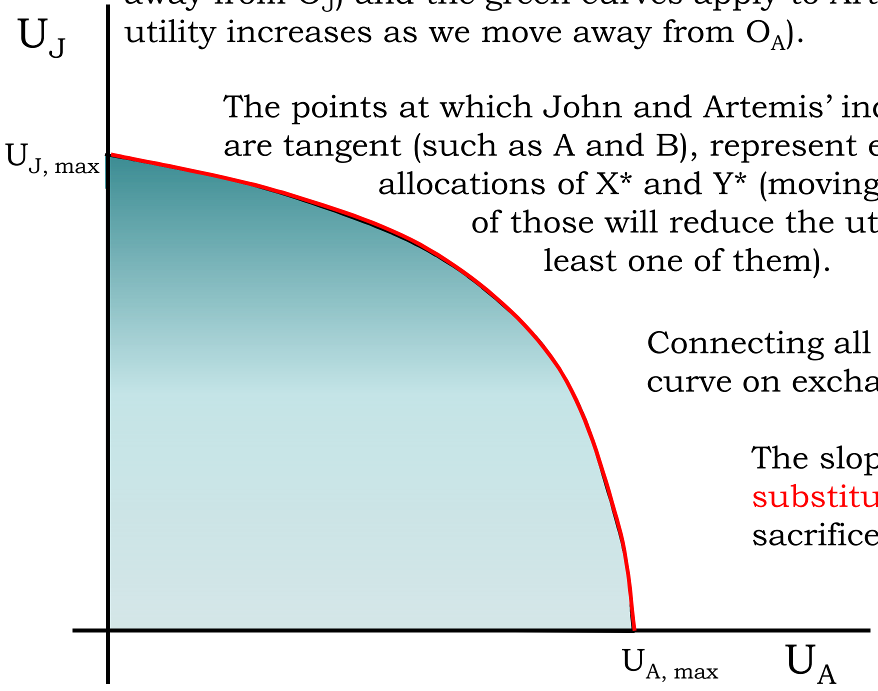
The next step is to find the efficient allocation of the two goods produced to the two individuals (Artemis and John).

Assuming we choose a point on the contract curve that produced X* and Y* we can create an Edgeworth box. The top right-hand corner of the box represents the allocation in which John holds all the goods, while the bottom left corresponds to complete ownership by Artemis. Points within the box represent all possible distributions of these quantities to Artemis and John.




Individuals' behavior in the markets is determined by their indifference curves. The blue curves represent John's indifference curves (his utility increases as we move to curves further away from $O_J$) and the green curves apply to Artemis (her utility increases as we move away from $O_A$).

The points at which John and Artemis' indifference curves are tangent (such as A and B), represent efficient allocations of X* and Y* (moving away from any of those will reduce the utility of at least one of them).



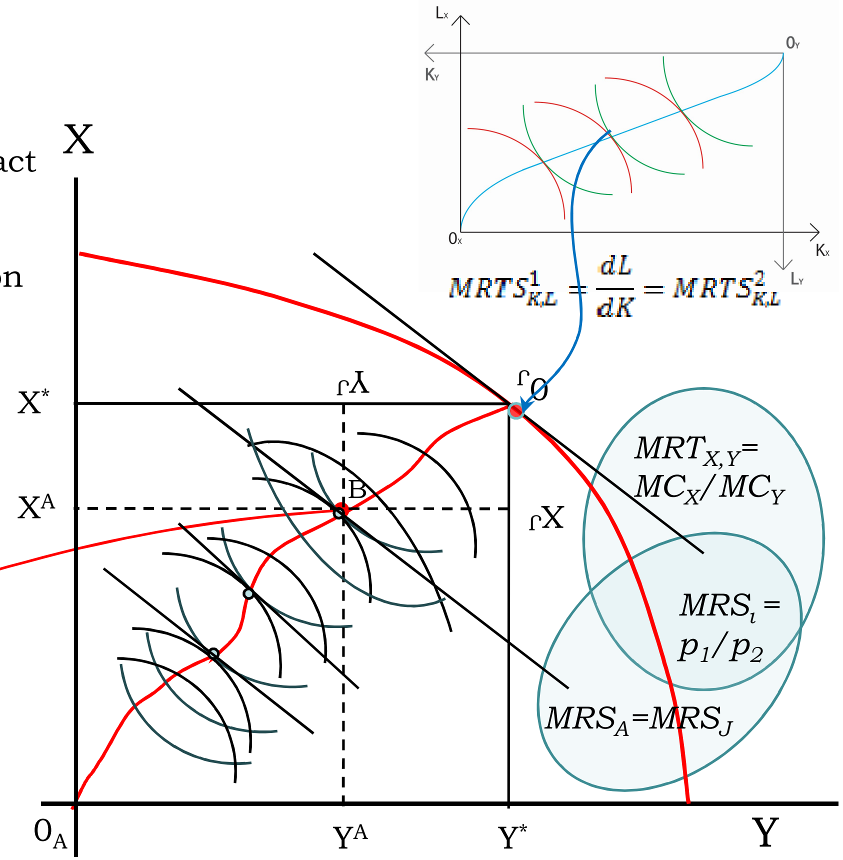

Connecting all tangency points (efficient allocations) we derive the contact curve on exchange (that gives us all efficient exchanges).

The slope of the indifference curves is called the marginal rate of substitution, MRS and gives the rate at which one individual is ready to sacrifice X in order to get more Y while keeping the same level of utility.

At the tangency points, the MRS of Artemis and John are equal, meaning that there are no further beneficial exchanges.

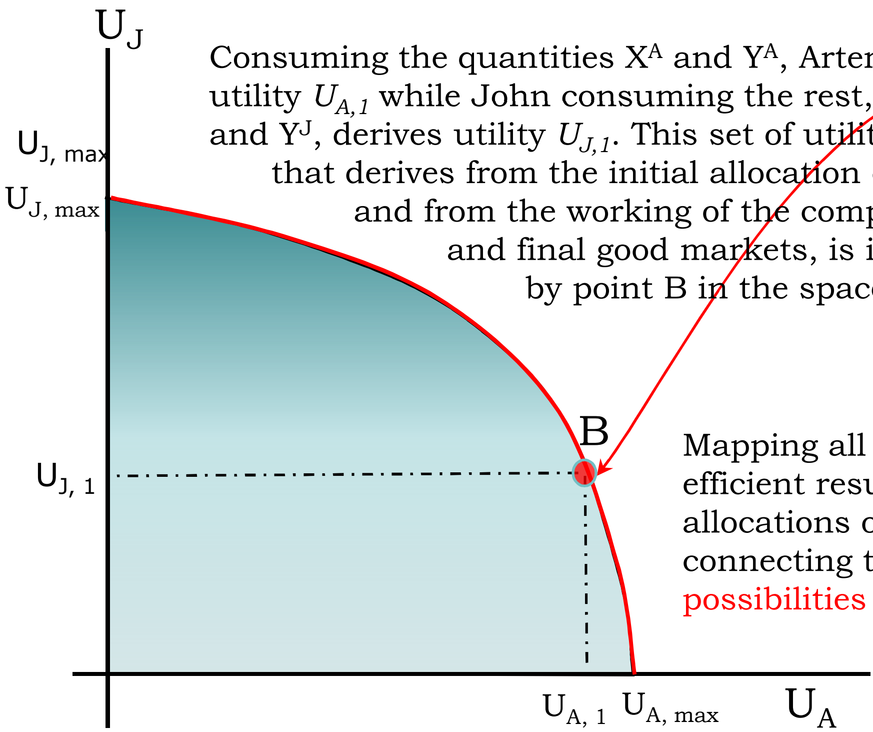
# Utility possibilities frontier

What facilitates the achievement of efficiency (in production and exchange) is competitive trading in the markets for inputs and final goods. Prices resulting from competitive trading are the signals guiding the allocation of resources along the input contract curve, and the exchange contract curve. Therefore, for any given initial allocation of resources between Artemis and John that is not Pareto efficient, competitive markets can lead to a reallocation that improves the utility of at least one of them.




Let us assume now an initial allocation that leads to the efficient allocation of production at point B on the exchange contract curve.

Consuming the quantities $X^A$ and $Y^A$, Artemis derives utility $U_{A,1}$ while John consuming the rest, that is $X^J$ and $Y^J$, derives utility $U_{J,1}$. This set of utility levels that derives from the initial allocation of resources and from the working of the competitive input and final good markets, is illustrated by point B in the space $U_A$, $U_J$.




Mapping all points that correspond to efficient results of all possible initial allocations of resources, and connecting them, yields the Utility possibilities frontier

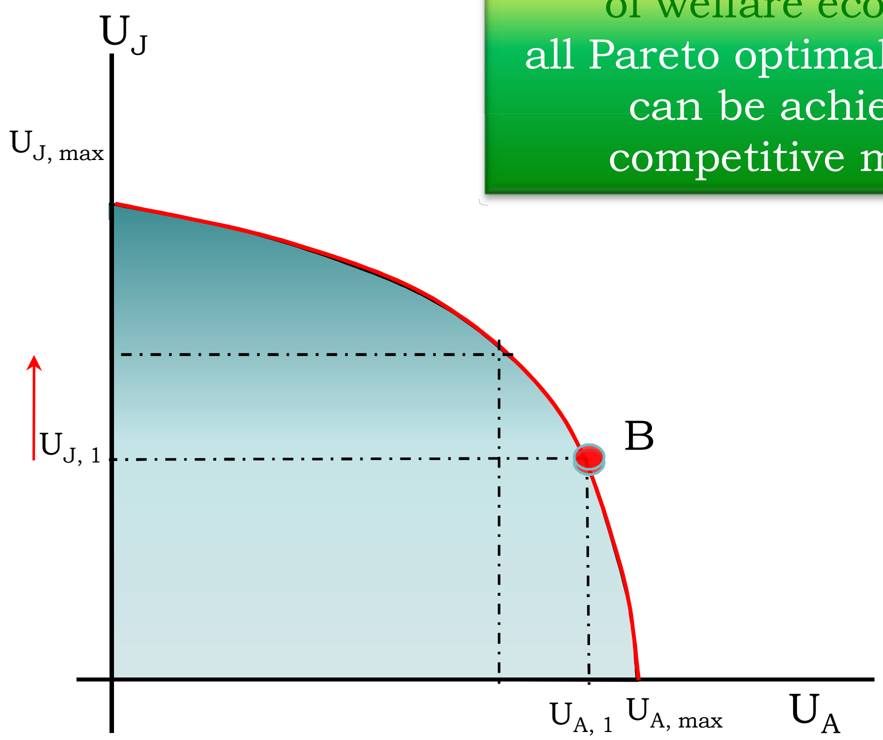
First fundamental Theorem of welfare economics: competitive markets result in Pareto optimal allocations for a given initial allocation

# Utility possibilities frontier

- The point is how you choose between points on the utility possibilities frontier.
- A social planner, who has the power, could actually redistribute property rights in such ways as competitive markets would lead to any possible point on the curve, i.e. to any possible allocation of utilities.
- It could therefore lead the society of Artemis and John to a fairer distribution.
- But how is defined what is fairer, and what expresses the will of society (collective preferences)?

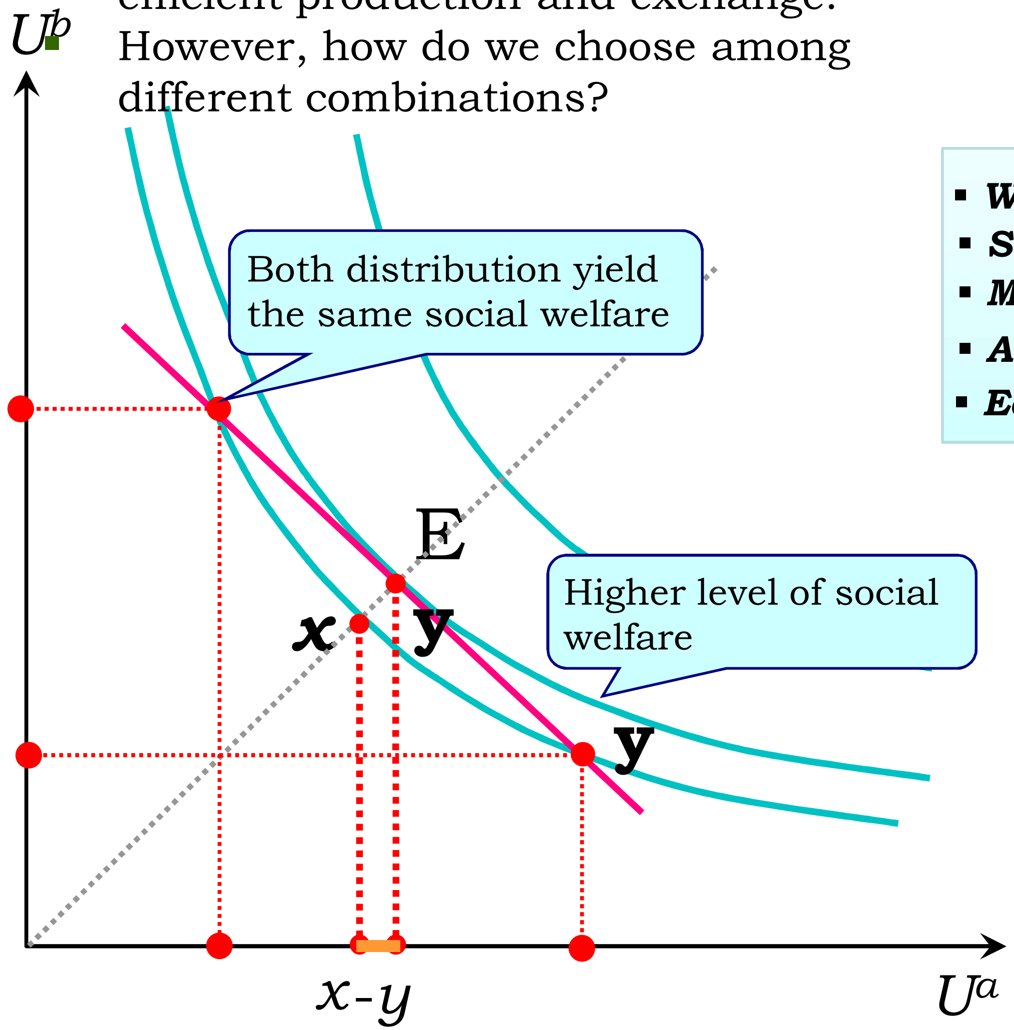
Second fundamental Theorem of welfare economics: all Pareto optimal allocations can be achieved by competitive markets.

The curve $U_{A,max}U_{J,max}$ illustrates the welfare of society, i.e. the maximum (efficient) combinations of the two society's members' utility that this society can achieve given the quantities of goods and services that can be produced with the given amounts of resources and technology.



# Social welfare indifference curves

- What economics can do well is, given an initial allocation of resources, to derive the maximum utilities for the members of the society, through competitive markets efficient production and exchange.
- However, how do we choose among different combinations?




- ***We start with a distribution at point y***
- ***Society's indifference curves W***
- ***Move to the exact opposite distribution***
- ***All distributions with the same mean***
- ***Equally-distributed-equivalent income***

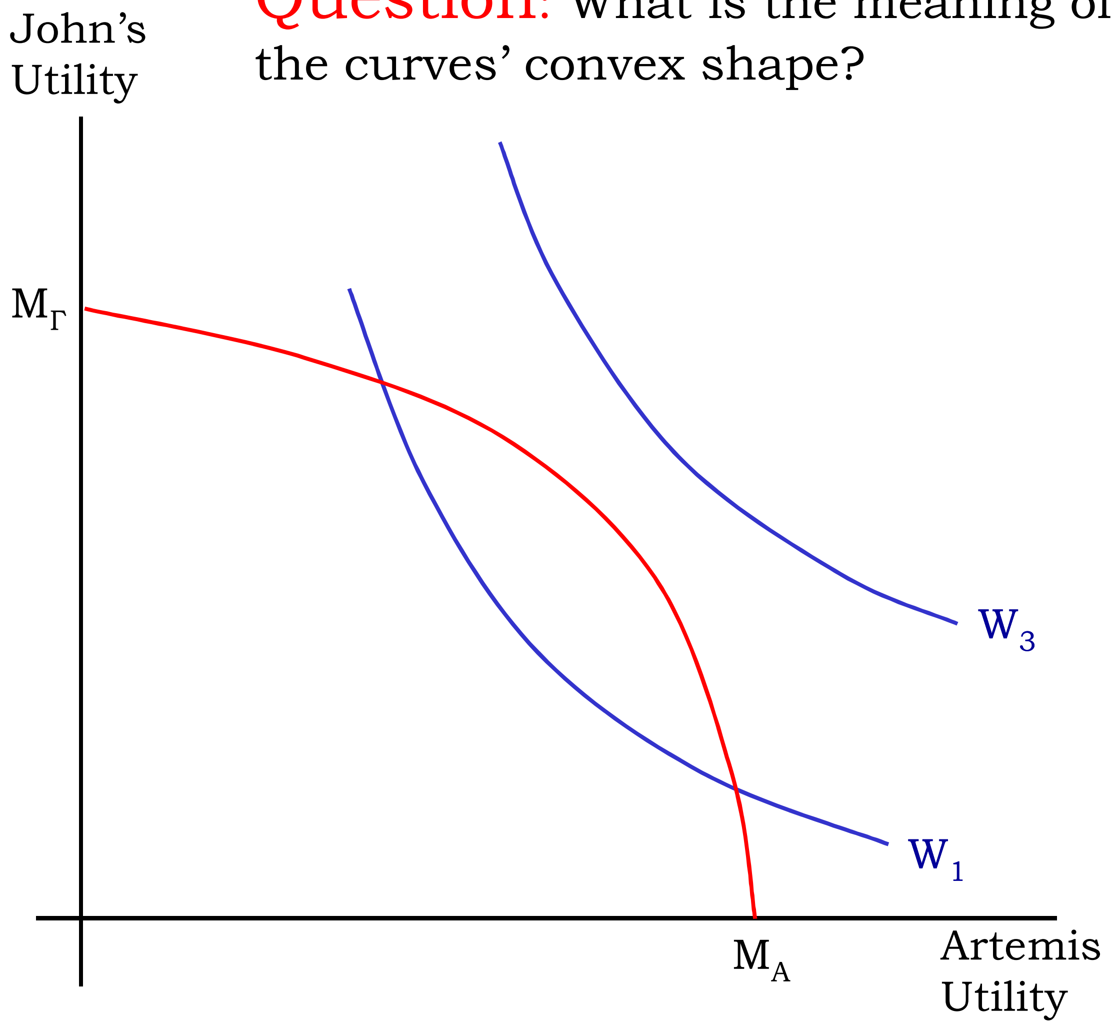
- *Anonymity implies symmetry W*
- at point E: *y* is the mean
- *Redistributions from the reach to the poor (from a to b) improve society's welfare W*
- x *is income that, if received uniformly by all, would yield same level of social welfare as* **y**
- E *y* –x *is income that society would give up to eliminate inequality*

# Social choice theory

- Assume that the social welfare function $W = W(U_\Gamma, U_A)$ yields society's indifference curves as those depicted in the graph.

Question: What is the meaning of the curves' convex shape?

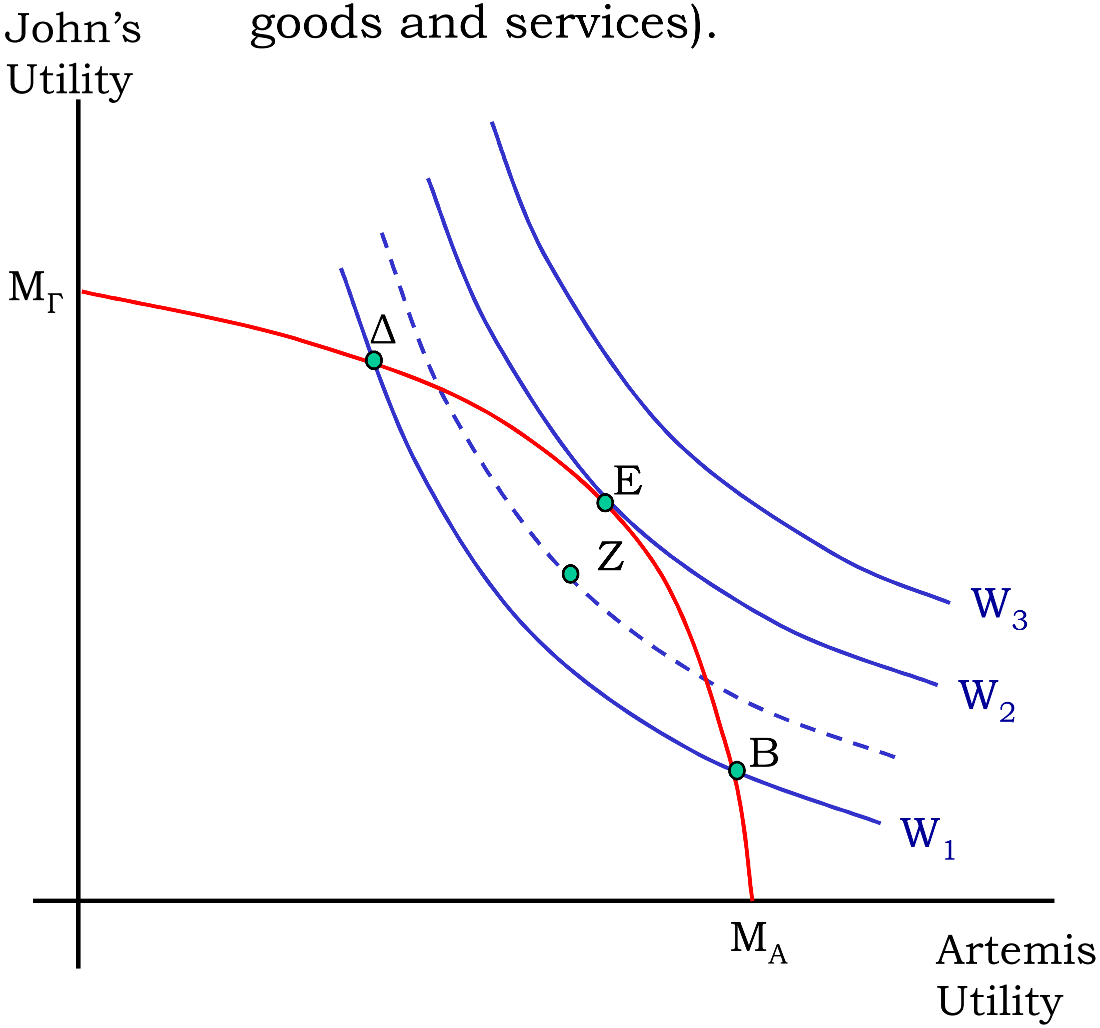
Declining rate of substitution between the utilities of John and Artemis. That is to say, they reflect a society that prefers equality between individuals (its willingness to put John (Artemis) in a better position than Artemis (John) is increasing as the well-being of John (Artemis) decreases).



# Social choice theory

- Due to the convex shape of the social indifference curves, combinations that are effective (on the $M_A M_\Gamma$ curve) such as those in points $B$ and $\Delta$ in the diagram, are considered by society to be inferior to even inefficient combinations, such as the one in point $Z$, but which result in more fair distributions.

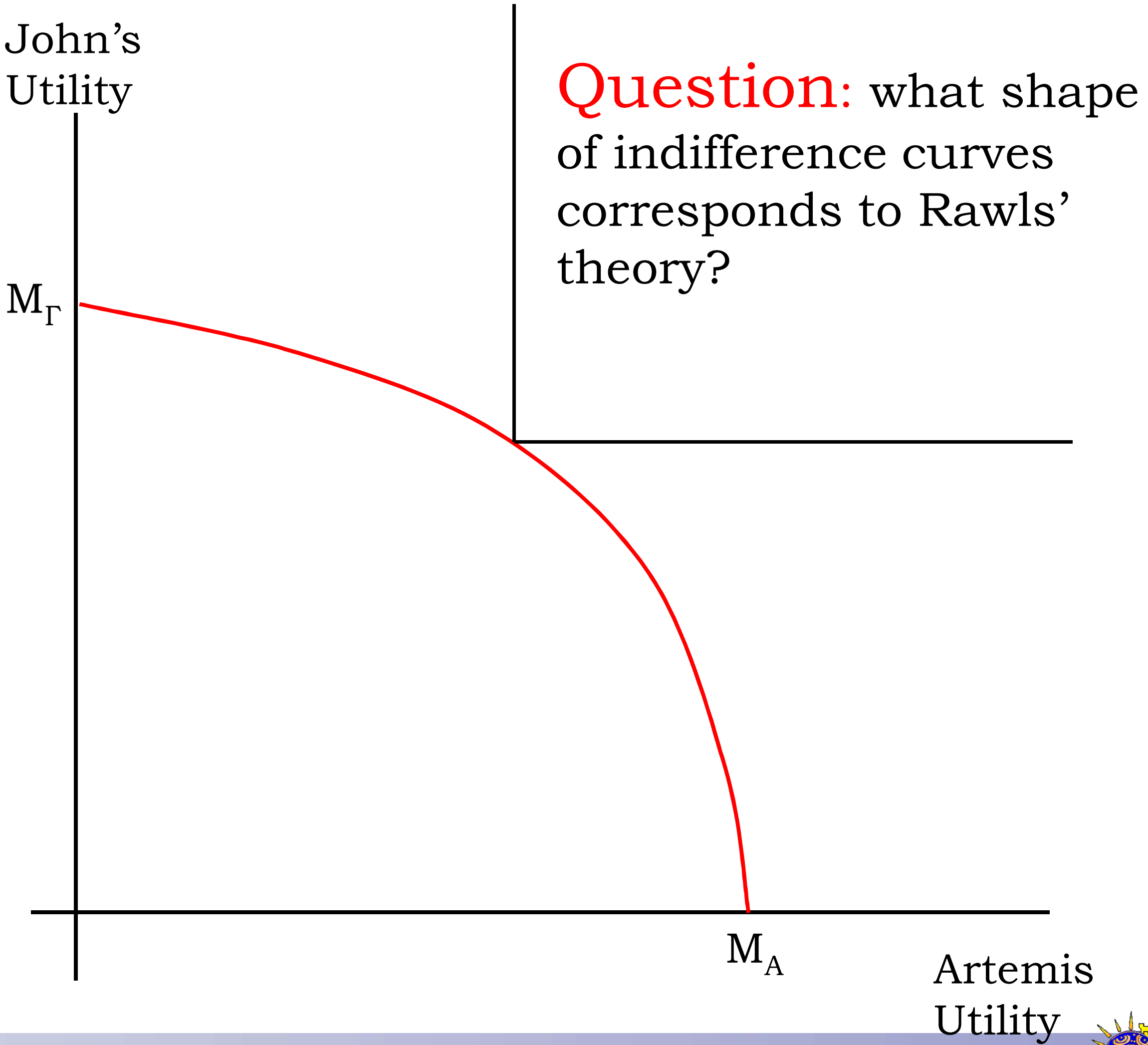
With these assumptions point $E$ of the diagram gives the excellent social distribution of utilities (and therefore goods and services).



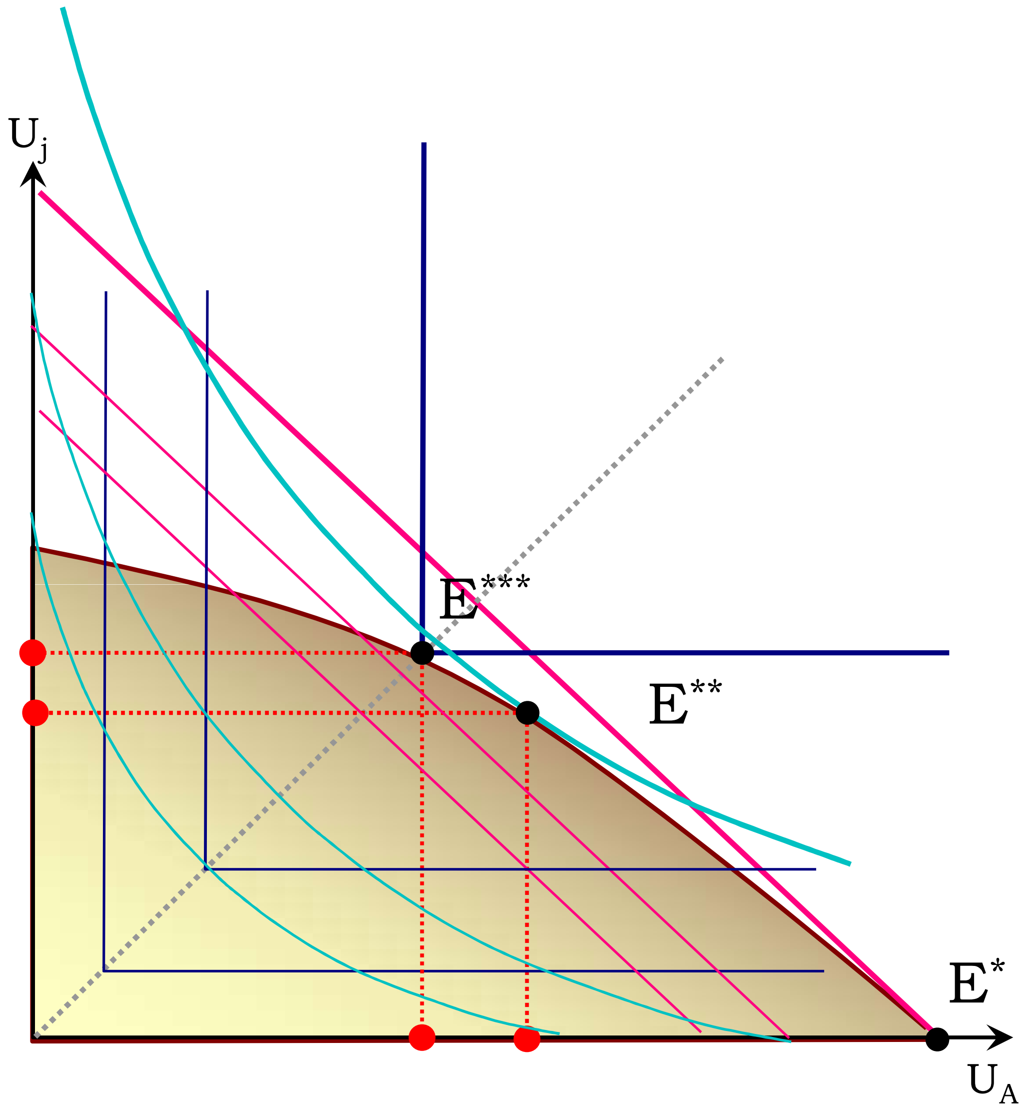
# Social choice theory

- However, there are many different shape that the social indifference curves can take, depending on the assumptions made regarding society's preferences

  - For example, the theory of J. Rawls, (veil of ignorance)
  - John Rawls suggests that if we didn't know our position regarding the initial distribution (we sit behind a veil of ignorance) then we would favor a much fairer social rule.



# Social choice theory




- *The utility-possibility set*
- *Welfare contours (*Utilitarian*)*
- *Welfare contours (*diminishing MRS*)*
- *Welfare contours (*Rawlsian*)*

- E* maximises total income irrespective of distribution
- E** trades off some income for greater equality
- E*** gives priority to equality; then maximises income subject to that

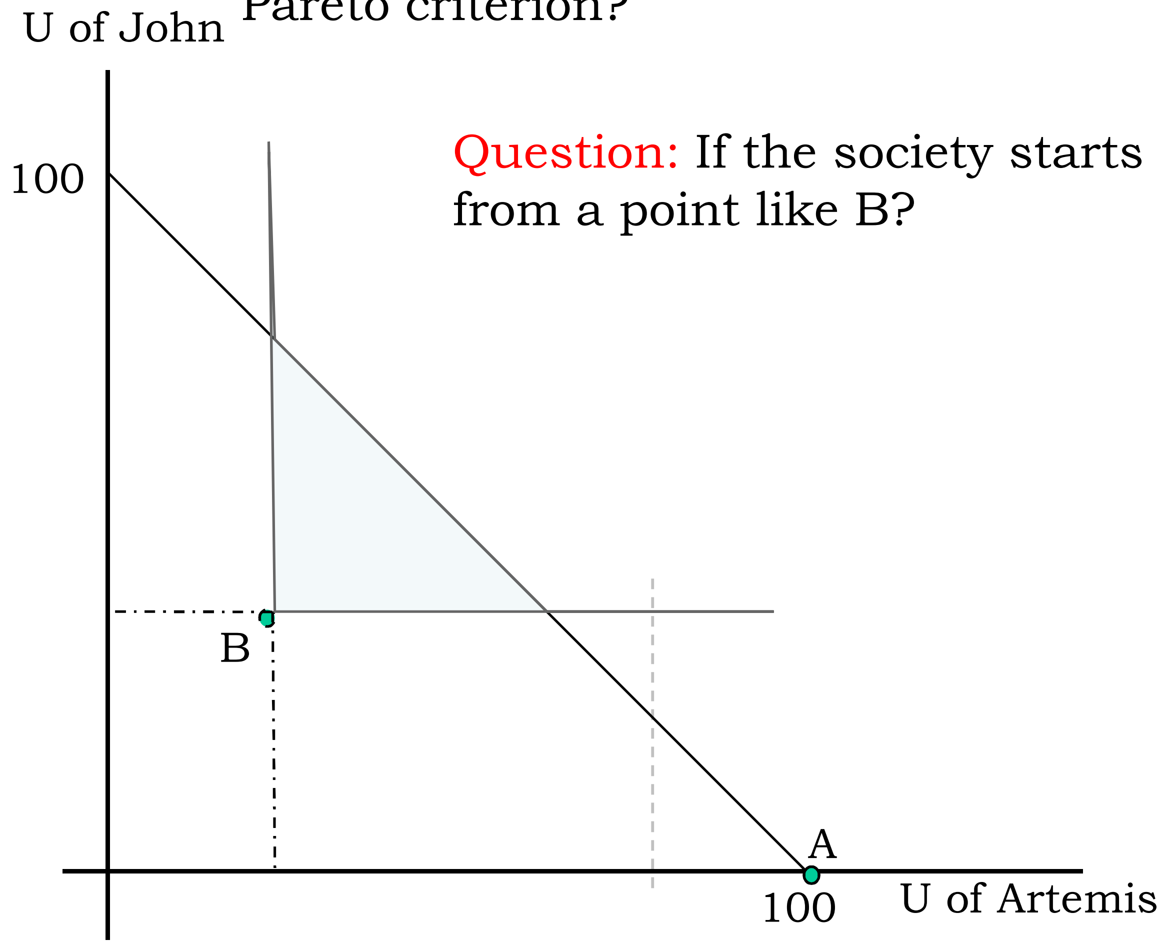
## Social choice theory

- As it is very difficult to determine the functional form of the social welfare, economic science has given greater weight to achieving efficiency.
- Economists usually resort to the safe harbor of the potential Pareto optimality criterion, to which we turn next.
- The strict Pareto efficiency criterion requires that the society approves a policy or a project that moves the economy from one distribution to another, only if the new distribution yields positive aggregate benefits and that a vector of transfers can be made so that at least one member of the society is better off without making any other member worse off.

# Social choice theory

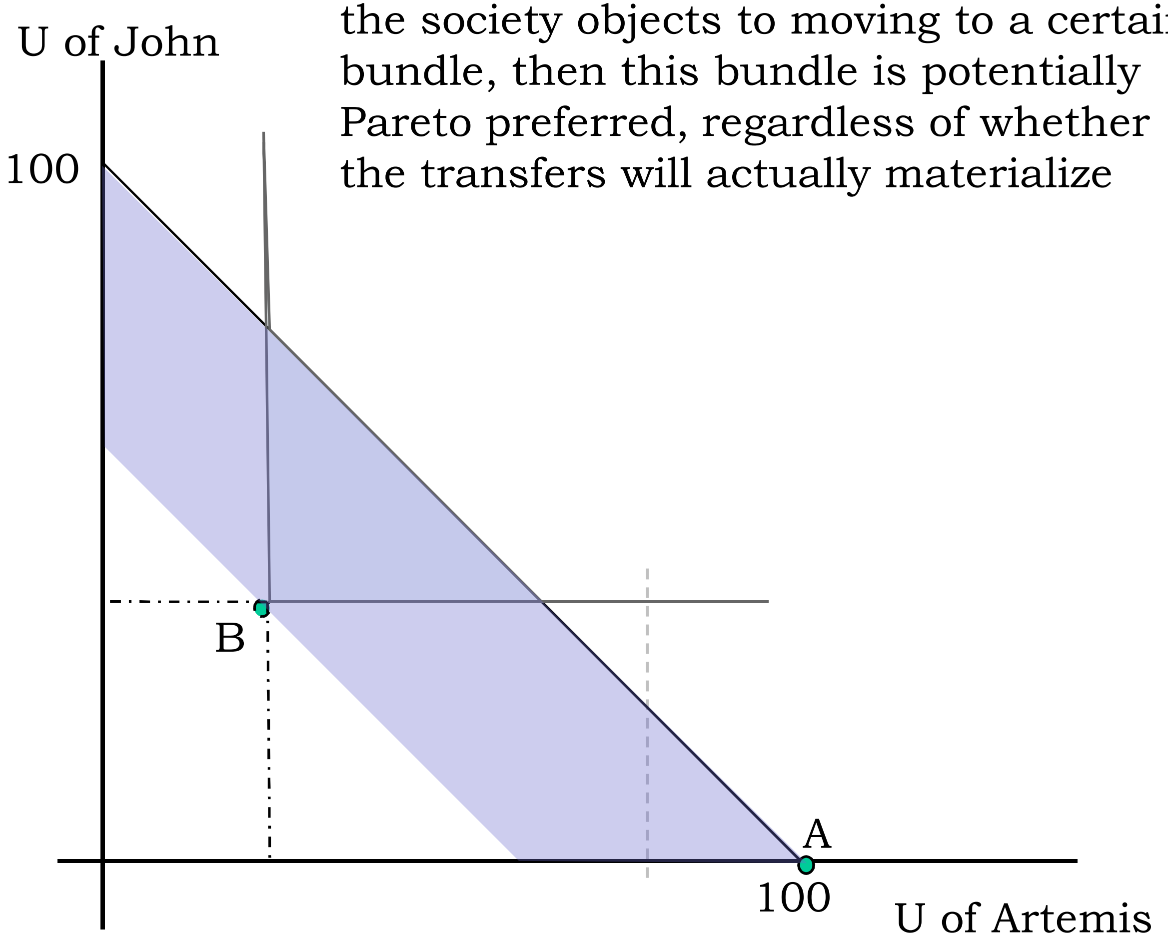
- Criticism: It is in general extremely difficult to find changes in allocation that comply with the Pareto criterion.

Question: If the society starts from an allocation indicated by point A (all resources are allocated to Artemis) which reallocations comply with the Pareto criterion?



# Social choice theory

Despite its appeal, the strict criterion poses serious limitations: Without transfers, the applicability of the criterion is very limited since policies that create only winners, and no losers are difficult to even imagine. Requiring all transfers to be made to potential losers, requires among others that all costs and benefits to each person affected by the policies are valued and costless institutions to administer the transfers are created. It is clear that most, if not all, policies would be unable to pass the requirements of the strict Pareto criterion and potential net benefits to society will be forgone.




An alternative is the so-called Kaldor-Hicks or potential Pareto criterion, according to which, if a vector of transfers exists, such that no member of the society objects to moving to a certain bundle, then this bundle is potentially Pareto preferred, regardless of whether the transfers will actually materialize

# Social choice theory

- The key difference between the strict Pareto efficiency and the Kaldor-Hicks criterion is that under the latter compensation to losers is in fact not paid. That is, the Kaldor-Hicks criterion in essence decouples considerations of efficiency from those of equity.

- Accordingly, it is clear that although every Pareto improvement is a Kaldor-Hicks improvement, not all Kaldor-Hicks improvements necessarily map onto Pareto improvements. The potential Pareto criterion provides the underlying rationale for the Cost Benefit Analysis.

- The potential Pareto criterion is importance in economics, as it has proven invaluable for comparing two distinct outcomes (with and without a policy/ project) while focusing only on market efficiency.

- With regards to distributional equity, researchers can still measure net benefits that accrue to different groups and present possible compensatory measures to policy makers.

- It should again be noted that there are other alternatives that could be used in devising choice criteria, including non-consequentialist theories such as the Rawlsian, briefly presented above, or libertarianism.