MICROECONOMICS II

Practice exercises - games of complete information

Problem 1: Consider a static game between two players as expressed by the payoff matrix below:

		Player 2		
		Left	**Center**	**Right**
Player 1	**Up**	**4, 1**	**4, 4**	**0, 5**
	Middle	**2, 1**	**2, 0**	**1, 2**
	Down	**3, 3**	**2, 1**	**0, 1**

a. What are the actions/strategies sets for the two players?
b. Is there any strictly dominated strategy for player 1? Is there any strictly dominated strategy for player 2? Explain your answer.
c. What is (are) the Nash Equilibrium (Equilibria) in pure strategies? Explain.
d. Is there a coordination failure in this game? Explain your answer.
e. What is the Nash Equilibrium in Mixed Strategies?

Problem 2: ***(Games with Positive Externalities)***

Two neighboring countries, $\boldsymbol{i = 1, 2}$, simultaneously choose how many resources (in hours) to spend in recycling activities, $\boldsymbol{r_i}$. The average benefit ($\boldsymbol{\pi_i}$) for every dollar spent on recycling is

$$\boldsymbol{\pi_i(r_i, r_j) = 10 - r_i + \frac{r_j}{2}}$$

and the (opportunity) cost per hour of recycling activity for each country is 4. Country $\boldsymbol{i}$'s average benefit is increasing in the resources that neighboring country $\boldsymbol{j}$ spends on recycling due to positive external effects on other countries.

a. Find each country's best-response function, and compute the Nash Equilibrium $\boldsymbol{(r_1^*, r_2^*)}$
b. Graph the best-response functions and indicate the pure strategy Nash Equilibrium on the graph.
c. On your previous figure, show how the equilibrium would change if the intercept of one of the countries' average benefit functions fell from 10 to some smaller number.

Problem 2': ***(Modifying the actions set in the above problem)***

Two neighboring countries, $\boldsymbol{i = 1, 2}$, simultaneously choose how many resources (in hours) to spend in recycling activities, $\boldsymbol{r_i}$. However, country 2 is facing a capacity constraint: $\boldsymbol{r_2}$ cannot exceed a certain maximum, say $\boldsymbol{r_{2MAX} = 3, 4}$. The average benefit ($\boldsymbol{\pi_i}$) for every dollar spent on recycling is

$$\boldsymbol{\pi_i(r_i, r_j) = 10 - r_i + \frac{r_j}{2}}$$

and the (opportunity) cost per hour of recycling activity for each country is 4. Country $\boldsymbol{i}$'s average benefit is increasing in the resources that neighboring country $\boldsymbol{j}$ spends on his recycling because a clean environment produces positive external effects on other countries.

a. Find each country's best-response function, and compute the Nash Equilibrium (r_1^*, r_2^*)
b. Graph the best-response functions and indicate the pure strategy Nash Equilibrium on the graph.
c. On your previous figure, show how the equilibrium would change if the cost per hour of recycling activity increases to 5.

Problem 2'': ***(Positive or negative externalities?)***

Two neighboring countries, $i = 1, 2$, simultaneously choose the level of recycling activities, r_i. The average benefit (π_i) on recycling is

$$\pi_i(r_i, r_j) = 10 - r_i + \frac{r_j}{2}$$

Country i's average benefit is increasing in the resources that neighboring country j spends on his recycling because a clean environment produces positive external effects on other countries. The recycling cost depends on the sum of recycling activities of the two countries and is it described by

$$C = \alpha(r_1 + r_2)^2$$

where $a \geq 0$.

a. Formally define the game.
b. Find each country's best-response function.
c. Compute the Nash equilibrium of this game.
d. How does the Nash Equilibrium change with changes in α?
e. Graph the best-response functions and indicate the pure strategy Nash Equilibrium on the graph.

Problem 3:

A luxury car manufacturer is producing "handmade" cars. The production of every single car requires assembling different parts, each prepared by a highly skilled worker. A car can only be produced if all necessary parts are available (i.e., car parts are perfect complements in production). The production level of the different parts depends on the effort a worker is putting on the job. Every worker $i \in \{1, 2, \dots, 10\}$ chooses an effort level $e_i \in [0, 1]$. The payoff to worker i is given by

$$\pi_i(e_1, \dots, e_{10}) = 5\, min\{e_1, \dots, e_{10}\} - 2e_i$$

a. Find all the Nash equilibria and prove that they are indeed Nash equilibria.
b. Are any of the Nash equilibria Pareto efficient? Justify your answer.
c. Suggest a mechanism to facilitate cooperation so that the Pareto Efficient Nash Equilibrium becomes the only equilibrium.

Problem 4:

Let there be two players, Anna and Beth, each having to put in an opaque jar either €0, or €400 or €700 of her own money. A third person, Charles, collects the money and counts the total amount, say X. Then Charles adds on X an amount of his own money according to the following rule:

- 0% of the total if $X \leq €400$
- 50% of the total if $€400 < X \leq €1100$
- 100% of the total if $X > €1100$

a. Finally, Charles splits the overall amount (i.e., the total AND his own contribution) equally between Anna and Beth. Anna and Beth care only about their net benefits.
b. Fully describe the game and construct the payoff matrix.
c. Is there a strictly dominant action for either player? Is there a strictly dominated action for either player? Fully explain your answer.
d. Find the Nash Equilibrium (or equilibria) of this game. Is there a problem with coordination failure?

Problem 5: (*exercise 35.2 in Osborne and Rubinstein*)
Two investors are involved in a competition with a prize of $1. Each investor can spend any amount in the interval $[0,1]$. The winner is the investor who spends the most; in the event of a tie each investor receives $0.50. Formulate this situation as a strategic game and find its mixed strategy Nash equilibria.

Problem 6: Litigation and Posner's nuisance rule. *Posner, R. 1997. Economic analysis of Law, 9th edition.*

ChemCo operates a chemical plant, which is located on the banks of a river. Downstream from the chemical plant is a group of fisheries. The ChemCo plant emits byproducts that pollute the river, causing harm to the fisheries. The profit ChemCo obtains from operating the chemical plant is $\$\boldsymbol{\Pi} > \mathbf{0}$. The harm inflicted on the fisheries due to water pollution is equal to $\$\boldsymbol{L} > \mathbf{0}$ of lost profit [without pollution the fisheries' profit is $\$\boldsymbol{A}$, while with pollution it is $\$(\boldsymbol{A} - \boldsymbol{L})$]. Suppose that the fisheries collectively sue the ChemCo Corporation. It is easily verified in court that ChemCo's plant pollutes the river. However, the values of $\boldsymbol{\Pi}$ and $\boldsymbol{L}$ cannot be verified by the court, although they are commonly known to the litigants. Suppose that the court requires the ChemCo attorney (Player 1) and the fisheries' attorney (Player 2) to play the following litigation game. Player 1 is asked to announce a number $\boldsymbol{x} \geq \mathbf{0}$, which the court interprets as a claim about the plant's profits. Player 2 is asked to announce a number $\boldsymbol{y} \geq \mathbf{0}$, which the court interprets as the fisheries' claim about their profit loss. The announcements are made simultaneously and independently. Then the court uses Posner's nuisance rule to make its decision. According to the rule, if $\boldsymbol{y} > \boldsymbol{x}$, then ChemCo must shut down its chemical plant. If $\boldsymbol{x} \geq \boldsymbol{y}$, then the court allows ChemCo to operate the plant, but the court also requires ChemCo to pay the fisheries the amount $\boldsymbol{y}$. Note that the court cannot force the attorneys to tell the truth: in fact, it would not be able to tell whether the lawyers were reporting truthfully. Assume that the attorneys want to maximize the payoff (profits) of their clients.
(a) Represent this situation as a normal-form game by describing the strategy set of each player and the payoff functions.
(b) Is it a dominant strategy for the ChemCo attorney to make a truthful announcement (i.e., to choose $\boldsymbol{x} = \boldsymbol{\Pi}$)?
(c) Is it a dominant strategy for the fisheries' attorney to make a truthful announcement (i.e., to choose $\boldsymbol{y} = \boldsymbol{L}$)?
(d) For the case where $\boldsymbol{\Pi} > \boldsymbol{L}$, find all the Nash equilibria of the litigation game.
(e) For the case where $\boldsymbol{\Pi} < \boldsymbol{L}$, find all the Nash equilibria of the litigation game.
(f) Does the court rule give rise to a Pareto efficient outcome?

<u>Problem 7</u>: Final-offer arbitration *Farber, H. 1980. "An Analysis of Final-Offer Arbitration." Journal of Conflict Resolution 35:683-705.*

Suppose there is a dispute between a firm and a union concerning wages. Let the timing of the game be as follows. First, the firm and the union simultaneously make offers, denoted by $\boldsymbol{w_f}$ and $\boldsymbol{w_u}$, respectively. Second, an arbitrator chooses one of the two offers as the settlement. Assume that the arbitrator has an ideal settlement she would like to impose, denoted by $\boldsymbol{x}$. Assume further that, after observing the parties' offers, $\boldsymbol{w_f}$ and $\boldsymbol{w_u}$, the arbitrator simply chooses the offer that is closer to $\boldsymbol{x}$: provided that $\boldsymbol{w_f} < \boldsymbol{w_u}$, (as is intuitive, and will be shown to be true), the arbitrator chooses $\boldsymbol{w_f}$ if $\boldsymbol{x} < (\boldsymbol{w_f} + \boldsymbol{w_u})/\boldsymbol{2}$, and chooses $\boldsymbol{w_u}$ if $\boldsymbol{x} > (\boldsymbol{w_f} + \boldsymbol{w_u})/\boldsymbol{2}$. The arbitrator knows $\boldsymbol{x}$ but the parties do not. The parties believe that $\boldsymbol{x}$ is randomly distributed according to a cumulative probability distribution denoted by $\boldsymbol{F}(\boldsymbol{x})$, with associated probability density function denoted by ***f(x)***. What is the pure strategy Nash equilibrium?

<u>Problem 8</u>: A location problem *Hotelling, H. 1929. "Stability in Competition." Economic Journal 39:41-57.*

Consider a population of voters uniformly distributed along the ideological spectrum from left ($\boldsymbol{x} = \boldsymbol{0}$) to right ($\boldsymbol{x} = \boldsymbol{1}$). Each of the candidates for a single office simultaneously chooses a campaign platform (*i.e.*, a point on the line between $\boldsymbol{x} = \boldsymbol{0}$ and $\boldsymbol{x} = \boldsymbol{1}$). The voters observe the candidates' choices, and then each voter votes for the candidate whose platform is closest to the voter's position on the spectrum. If there are two candidates and they choose platforms $\boldsymbol{x_1} = \boldsymbol{0.3}$ and $\boldsymbol{x_2} = \boldsymbol{0.6}$, for example, then all voters to the left of $\boldsymbol{x} = \boldsymbol{0.45}$ vote for candidate 1, all those to the right vote for candidate 2, and candidate 2 wins the election with 55 percent of the vote. Suppose that the candidates care only about being elected—they do not really care about their platforms at all! If there are two candidates, what is the pure strategy Nash equilibrium?