GAME THEORY

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LECTURE NOTES SET 5: REPEATED GAMES

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Repeated Games

What if a static game* is played again (repeated) once it is over?

If a game is played repeatedly, with the same players who know each other's previous moves, the players may behave very differently than if the game is played just once (a one-shot game).

- **Is it possible that future promises/threats can affect/alter the outcome of the game?**
  - Cooperation in a current stage can be induced in some cases through promises and threats provided some conditions…
- **How do strategies in the repeated game look like?**
  - Players' behavior can be conditioned on the "history" of the game → credibility of threats (?)
- **Can we represent graphically a repeated game with a game-tree?**
  - We can do that but it gets really complicated really fast (just with few repetitions)
- **What is the appropriate solution concept?**
  - Since the game is evolved over time (i.e., stages are taking place sequentially) the proper solution concept is Subgame Perfect Nash Equilibrium (SPNE)

* The concept of repeated games can very well apply to extensive form games. With some differences in the way strategies are defined, the main results of the analysis that follow holds true for these cases as well.

Repeated Games

Characteristics of a repeated game:

- Identical game played in every stage.
- Payoffs are realized at the end of every stage and sequentially added until the end of the repeated game.
- Payoffs from one stage do not directly affect payoffs of another stage.
- Players know each other's previous moves.

Two types of repeated games

- **Finitely repeated:** the game is played for a finite and known number of rounds/stages.
- **Infinitely or Indefinitely repeated:** the game has no predetermined length; players act as though it will be played indefinitely, or it ends only with some probability.

The outcome of a repeated game can greatly change depending on whether it is finitely or infinitely repeated!

Finitely Repeated Games

Example 1

Consider a typical "prisoner's dilemma" game as in the table below:

		Player 2	
		Lie	Confess
Player 1	Lie	-1, -1	-9, 0
	Confess	0, -9	-6, -6

This game, as we know, has a unique Nash Equilibrium, namely (Confess, Confess).

What will happen if this game is repeated twice? Can the cooperative outcome (-1, -1) be somehow supported?

- Player 1 can, for example, ask player 2 to lie in the first stage and in exchange she will lie both in the first and in the second stage. Is this subgame perfect?
  - Consider the last stage: since it is the last time this game is played, both players will adopt their dominant strategy, i.e., to Confess
  - Any promise/threat about Stage 2's action different than Confess is not credible! In stage 2, the only subgame Nash Equilibrium is the unique Nash Equilibrium of the game!
  - Hence, in the first stage the only possible equilibrium (given that no cooperation can take place in the second stage) is for both to confess!

Finitely Repeated Games

The previous example reveals two important results that can be generalized for finitely repeated games:

Proposition 1

In the final subgame players must play their action that corresponds to a Nash Equilibrium in all Subgame Perfect Nash Equilbria.

- All payoffs from before cannot change
- The Nash equilibrium Strategy is the only strategy that maximizes a player's payoffs in the last stage.

Proposition 2 (Selten)

If the stage game has a unique Nash Equilibrium then for any finite number of repetitions the repeated game has a unique SPNE: the players play the actions that correspond to the unique Nash equilibrium in every stage of the game.

Finitely Repeated Games

One more significant result about repeated games is the following:

Proposition 3

A collection of Nash Equilibrium strategies in every stage of the game is indeed a Subgame Perfect Nash Equilbrium

- When choosing the Nash Equilibrium strategy future strategies cannot be affected
- The Nash equilibrium Strategy is the only strategy that maximizes a player's payoffs in the future.

Very Important: the above proposition does not describe all Subgame Perfect Nash Equilibria! There could be more, where players use strategies that contain actions-responses to previous plays! Cooperation, to some extend, may be possible!

Finitely Repeated Games

Example 2

Consider a static game described by the table below:

		Player 2		
		Left	Center	Right
Player 1	Up	1, 1	5, 0	0, 0
	Middle	0, 5	4, 4	0, 0
	Down	0, 0	0, 0	3, 3

This game has two Nash Equilibria, namely (Up, Left) and (Down, Right). Moreover, there is coordination failure as none of these N.E. is Pareto Efficient.

What is (are) the SPNE of this game?

Finitely Repeated Games

… Example 2

Based on Proposition 1 we can infer that, in the last stage, players will either end up in (D, R) or in (U, L).

- Assume that in the last stage the payers end up in (U, L). Then, at the beginning of the first stage the payoff matrix looks like

		Player 2		
		Left	Center	Right
Player 1	Up	1+1, 1+1	5+1, 0+1	0+1, 0+1
	Middle	0+1, 5+1	4+1, 4+1	0+1, 0+1
	Down	0+1, 0+1	0+1, 0+1	3+1, 3+1

		Player 2		
		Left	Center	Right
Player 1	Up	2, 2	6, 1	1, 1
	Middle	1, 6	5, 5	1, 1
	Down	1, 1	1, 1	4, 4

**** we have added in every cell the payoffs that correspond to the N.E. (U, L) of the last stage!*

Finitely Repeated Games

… Example 2

We now look for Nash Equilibria in the new payoff matrix. The resulting Nash Equilibria can be supported as SPNEs provided some proper strategies!

		Player 2		
		Left	Center	Right
Player 1	Up	2, 2	6, 1	1, 1
	Middle	1, 6	5, 5	1, 1
	Down	1, 1	1, 1	4, 4

This game supports two SPNEs:

#1 play U in the 1st period and play U in the 2nd period no matter what

#2 play L in the 1st period and play L in the 2nd period no matter what

#1 play D in the 1st period and play U in the 2nd period no matter what

#2 play R in the 1st period and play L in the 2nd period no matter what

Finitely Repeated Games

… Example 2

- Assume that in the last stage the payers end up in (D, R). Then, at the beginning of the first stage the payoff matrix looks like

		Player 2		
		Left	Center	Right
Player 1	Up	1+3, 1+3	5+3, 0+3	0+3, 0+3
	Middle	0+3, 5+3	4+3, 4+3	0+3, 0+3
	Down	0+3, 0+3	0+3, 0+3	3+3, 3+3

		Player 2		
		Left	Center	Right
Player 1	Up	4, 4	8, 3	3, 3
	Middle	3, 8	7, 7	3, 3
	Down	3, 3	3, 3	6, 6

**** we have added in every cell the payoffs that correspond to the N.E. (D, R) of the last stage!*

Finitely Repeated Games

… Example 2

We now look for Nash Equilibria in the new payoff matrix. The resulting Nash Equilibria can be supported as SPNEs provided some proper strategies!

		Player 2		
		Left	Center	Right
Player 1	Up	4, 4	8, 3	3, 3
	Middle	3, 8	7, 7	3, 3
	Down	3, 3	3, 3	6, 6

This game supports two SPNEs:

#1 play U in the 1st period and play D in the 2nd period no matter what

#2 play L in the 1st period and play R in the 2nd period no matter what

#1 play D in the 1st period and play D in the 2nd period no matter what

#2 play R in the 1st period and play R in the 2nd period no matter what

Finitely Repeated Games

… Example 2

In this the end?

		Player 2		
		Left	Center	Right
Player 1	Up	1, 1	5, 0	0, 0
	Middle	0, 5	$\boxed{4, 4}$	0, 0
	Down	0, 0	0, 0	3, 3

Is there any way to support the Pareto Efficient outcome of (M, C) at least for the first period?

- proposition 1 makes clear that this outcome cannot be supported for the last period, but no one says it cannot be supported for a period before the last!
- It turns out that outcomes other than those corresponding to N.E. can indeed be supported in previous stages provided a proper mechanism of rewards and punishments!

Finitely Repeated Games

… Example 2

- It is possible that the players will anticipate that different first stage outcomes are linked with different Nash Equilibria of the last stage! For example, assume that players anticipate the outcome of (D, R) in the last stage only if the outcome of (M, C) results in the first stage. In any other case, the last stage outcome will be the one corresponding to (U, L). Then the payoff matrix is transformed as follows:

		Player 2		
		Left	Center	Right
Player 1	Up	1+1, 1+1	5+1, 0+1	0+1, 0+1
	Middle	0+1, 5+1	4+3, 4+3	0+1, 0+1
	Down	0+1, 0+1	0+1, 0+1	3+1, 3+1

		Player 2		
		Left	Center	Right
Player 1	Up	2, 2	6, 1	1, 1
	Middle	1, 6	7, 7	1, 1
	Down	1, 1	1, 1	4, 4

Finitely Repeated Games

… Example 2

This game supports three SPNEs:

#1 play U in the 1st period and play U in the 2nd period no matter what

#2 play L in the 1st period and play L in the 2nd period no matter what

#1 play D in the 1st period and play U in the 2nd period no matter what

#2 play R in the 1st period and play L in the 2nd period no matter what

		Player 2		
		Left	Center	Right
Player 1	Up	2, 2	6, 1	1, 1
	Middle	1, 6	7, 7	1, 1
	Down	1, 1	1, 1	4, 4

#1 play M in the 1st period; play D in the 2nd period if (M, C) is observed in the 1st period, otherwise play U

#2 play C in the 1st period; play R in the 2nd period if (M, C) is observed in the 1st period, otherwise play L