GAME THEORY

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LECTURE NOTES SET 4: EXTENSIVE FORM GAMES

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PART II: EXTENSIVE FORM (OR DYNAMIC) GAMES OF PERFECT INFORMATION

In many cases of conflict time is a crucial element. Unlike static games where actions have to be taken simultaneously, in **extensive form games** players choose their actions sequentially according to a specific order (i.e., a "protocol").

> *An extensive game is a detailed description of the sequential structure of the decision problems encountered by the players in a strategic interaction.*
>
> *Osborne and Rubinstein (1994)*

Example 1: Prisoner's dilemma with a twist

Consider again the prisoner's dilemma game with a twist: in a first stage prisoner 1 has to decide whether to confess or lie; in a second stage, prisoner 2 observes what prisoner 1 has done in the previous stage and will decide whether to confess or lie. Payoffs are realized at the end of stage 2 according to the payoff matrix of the game we 've seen before:

		Player 2	
		Lie	Confess
Player 1	Lie	-1, -1	-9, 0
	Confess	0, -9	-6, -6

- What do you think will be the outcome of this situation?
- Can you imagine a different solution than the one in the static counterpart of the game?

Example 2: Battle of sexes with a twist

Consider again the battle of sexes game with a twist: in a first stage Mary has to decide whether to go to the Opera or the Fight; in a second stage, Peter observes what Mary has done in the previous stage and will decide whether to go to the Opera or to the Fight. Payoffs are realized at the end of stage 2 according to the payoff matrix of the game we ‘ve seen before:

		Player 2	
		UFC	Opera
Player 1	UFC	2, 1	0, 0
	Opera	0, 0	1, 2

- What do you think will be the outcome of this situation?

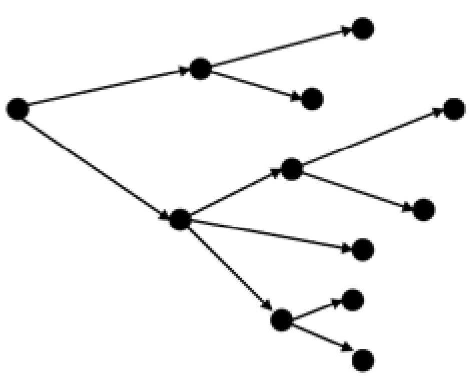
Representation of extensive form games

We are using **game-trees** to represent this kind of games. A game-tree is a set of **nodes** and **directed edges** connecting these nodes such that

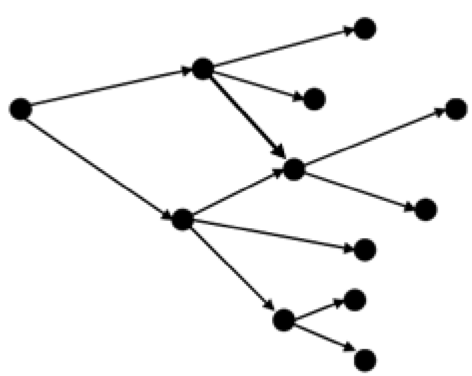
- there is only one initial node for which there is no incoming edge
- for every other node there is exactly one incoming edge
- for any two nodes that are not successors of each other, the union of these nodes' successors is the empty set
- nodes that are not followed by another node are terminal nodes

For example,

A game-tree



Not a game-tree



Solution concept of extensive form games

In extensive form games (of complete and perfect information) **credibility** is a central issue: the player who follows can only commit to actions that are best responses to the choices of the player that precedes. Intuitively, every player anticipates the optimal actions that players acting in subsequent stages will select, and chooses his actions accordingly (**sequentially rational actions**)

- For example, in the battle of the sexes game IF Mary moves first and decide to go the fight, Peter has no means to make her change her mind: Peter threatening to go to the Opera while Mary is at the Fight is a non-credible threat since going to the Fight will be Peter's best response when Mary is at the Fight!
  - The typical Nash equilibrium that is used as a solution concept in static games is not sufficient as it does not exclude non-credible threats!

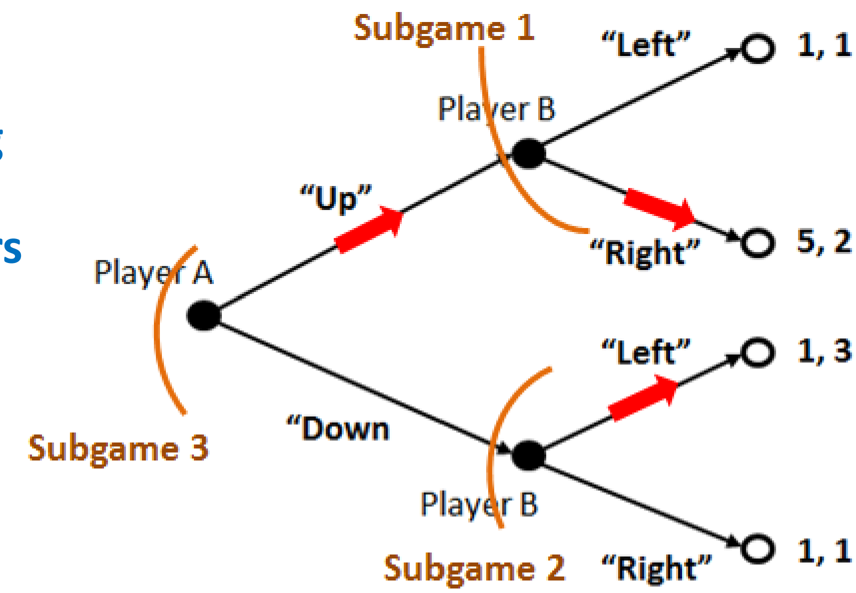
The solution concept we apply to extensive form games of complete information is that of **Subgame Perfect Equilibrium**, SPE (or, **Subgame Perfect Nash Equilibrium**, SPNE). We are using **Backward Induction** to solve these games

... Solution concept of extensive form games

Example

We start from the last stage:

- (subgame 1) if player A had played "Up" then player 2 compares moving "Left" and getting 1 to moving "Right" and getting 2. Player B prefers 2 than 1 so he is choosing "Right"
- (subgame 2) if player A had played "Down" then player 2 compares moving "Left" and getting 3 to moving "Right" and getting 1. Player B prefers 3 than 1 so he is choosing "Left"
- (subgame 3) In the first stage player A knows how player B will respond to his choices. Hence, player A knows that should he play "Up" he will receive 5 (since player B responds to "Up" by "Right") while should he play "Down" he will receive 1 (since player B responds to "Down" by "Left")



****Actions vs. Strategies**

- The set of actions for the two players are $A_1 = \{U, D\}$ and $A_2 = \{R, R', L, L'\}$. The set of strategies of the two players are $S_1 = \{U, D\}$ and $S_2 = \{RL', RR', LL', LR'\}$

Solution concept of extensive form games

Definition: To fully define Extensive form game the followings must be defined

- $\mathcal{N} = \{1, 2, \dots, n\}$ players
- $X = \{v_0, v_1, \dots, v_m\}$ is the set of nodes and it is partitioned in $n + 1$ non-joint sets

Also let

- X_i the set of nodes belonging to player $i \in \mathcal{N}$, where $X_i \subset X$
- $X(v_l)$ is the set of successors of v_l

If $v_l \in X$ but $v_l \notin X_i$ for all $i \in \mathcal{N}$, then v_l is a terminal node (*i.e.*, a node that it does not belong to any player). In addition v_0 is the starting node.

- For a non-terminal node v_l that belongs to a player i, $A_{v_l} = \{a_{v_l}^1, a_{v_l}^2, \dots, a_{v_l}^k\}$ is the set of actions of player i at the node $v_l \in X_i$
- $S_i = \{s_i^1, s_i^2, \dots, s_i^m\}$ is the set of strategies of player i

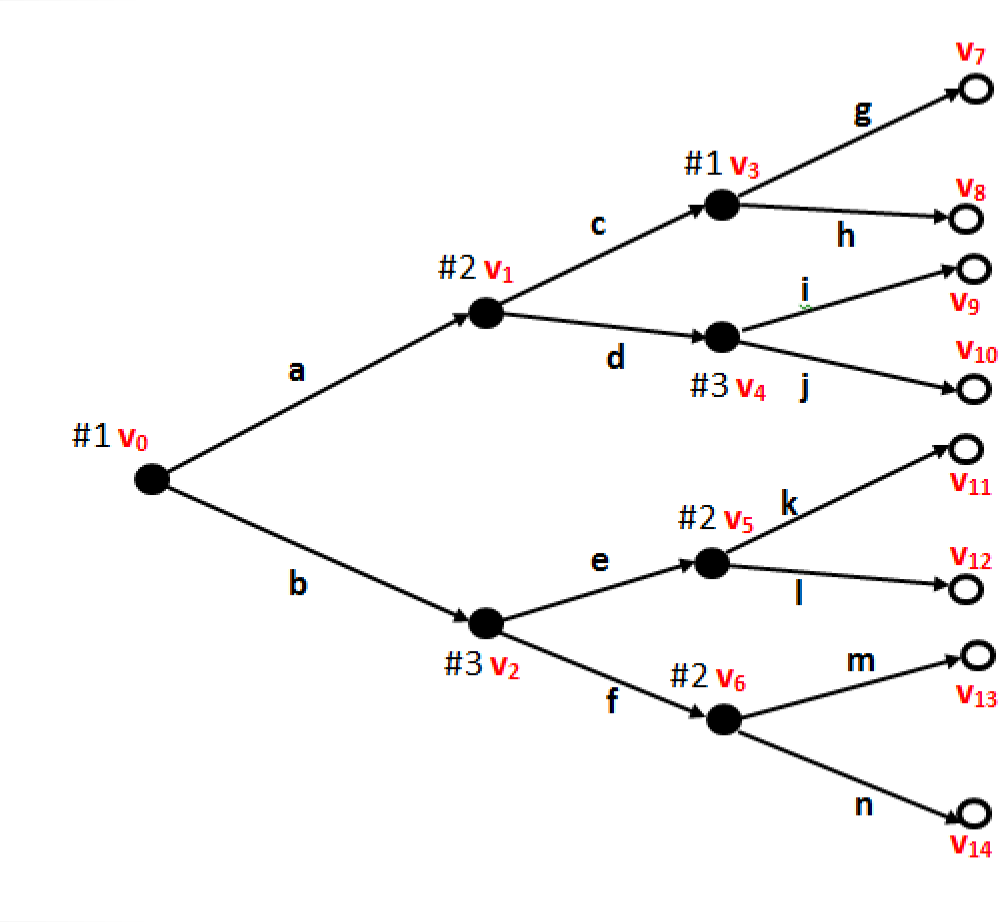
A strategy of player is a function $s_i: X_i \rightarrow A_i$ such that $s_i(v_l) \in A_{v_l}$ (*i.e.*, the strategy assigns an action of player i to every node that belongs to her).

- $u_i: S \rightarrow \mathcal{R}$, where $S = \times_{i \in \mathcal{N}} S_i$ is the payoff function of player i

Starting from any node a subgame consists of $X(v_l)$

Example

Properly describe the game represented by the game-tree below:



Example 3: Pareto coordination game with a twist

Consider again the Pareto coordination game with a twist: in a first stage Firm 1 has to decide whether to adopt technology A or B; in a second stage, firm 2 observes what firm 1 has done in the previous stage and will decide whether to adopt technology A or B. Payoffs are realized at the end of stage 2 according to the payoff matrix of the game we 've seen before:

		Firm 2	
		Tech. A	**Tech. B**
Firm 1	**Tech. A**	10, 8	0, 0
	Tech. B	0, 0	6, 5

a) Draw the extensive form game
b) What do you observe?

Example 4: Matching the pennies game with a twist

Consider again the matching the pennies game with a twist: in a first stage player 1 has to decide whether to place his coin facing up (i.e., heads) or down (i.e., tails); in a second stage, player 2 observes what player 1 has done in the previous stage and will decide whether to place his coin facing up (i.e., heads) or down (i.e., tails). Payoffs are realized at the end of stage 2 according to the payoff matrix of the game we 've seen before:

		Player 2	
		Heads	Tails
Player 1	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

a) Draw the extensive form game

b) What do you observe?

Practice problems

Practice with the following problems (ch. 4) from our textbook: