GAME THEORY

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LECTURE NOTES SET 3: MIXED STRATEGIES

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Mixed strategy: what is it?

In many games players choose unique actions from the set of available actions. These are called pure strategies. In some situations though a player may want to **randomise** over several actions. If a player is choosing which action to play randomly, we say that the player is using a "**mixed strategy**" as opposed to a pure strategy. In a pure strategy a player chooses an action for sure, whereas in a mixed strategy, she chooses a probability distribution over the set of actions available to her.

Mixed strategy: what is it?

Definitions

- A mixed strategy for player $\boldsymbol{i}$ is a probability distribution over her set of available actions. In other words, if player $\boldsymbol{i}$ has $\boldsymbol{m}$ actions available, a mixed strategy is an m-dimensional vector $(\boldsymbol{p1_i}, \boldsymbol{p2_i}, \ldots \boldsymbol{pm_i})$ such that $\boldsymbol{pk_i} \geq \boldsymbol{0}$ for all $\boldsymbol{k} = \boldsymbol{1}, \boldsymbol{2}, \ldots \boldsymbol{m}$, and $\sum_{\boldsymbol{k}=\boldsymbol{1}}^{\boldsymbol{m}} \boldsymbol{pk_i} = \boldsymbol{1}$

- A Mixed Strategy Nash Equilibrium (MSNE) is defined as the situation where player $\boldsymbol{i}$ chooses his probability distribution over his actions in order to make all other players $-\boldsymbol{i}$ indifferent among their choices, and that holds for all $\boldsymbol{i} \in \boldsymbol{N}$.

Mixed strategies: an example

The “penalty-kick” game

A goalkeeper is facing a Leo Messi who is about to kick a penalty shot. The goalie can leap to the left or the right while Messi can shoot left or right.* The assumptions here are that if the goalie leaps the to the opposite direction of the shot, the ball will hit the net 100% of the times. If, on the other hand, the goalie chooses his leap correctly (i.e., matching Messi’s shot) then the goalie will save 10% of the times if the shot is right and 40% of the time if the shot is left. The payoff matrix is as follows

		Goalie	
		Right	Left
Messi	Right	90, 10	100, 0
	Left	100, 0	60, 40

- This game has no pure strategy equilibrium.
- What if the goalie randomizes over his actions?

Mixed Strategy Nash Equilibrium (MSNE)

The concept of mixed strategies can be considered to be intuitively problematic. Why and how do players randomize their decisions?

- Randomizing over possible actions, a central concept in mixed strategies, lacks behavioral support. Seldom do people make their choices following a lottery.
- This behavioral problem is compounded by the cognitive difficulty that people are unable to generate random outcomes without the aid of a random or pseudo-random generator.

Then why to use them?

Mixed strategies are still widely used for their capacity to provide Nash equilibria in games where no equilibrium in pure strategies exists!

Mixed Strategy Nash Equilibrium (MSNE)

Rationalizing mixed strategies

- Mixed strategies being an object of choice: players DO randomize over their actions and this randomization is not ex-ante revealed
  - Think of tax authorities and taxpayers: random audits
    - The problem here is that the authorities would prefer to state the probability of an audit prior of playing the game so it is apparent to the taxpayers.
- Mixed strategies are formed based on beliefs of players about their rivals' behaviour
  - Plaisio discovers that in the past MediaMarkt had exclusive offers on tablets 8 out of 10 times in the month of March.
    - The problem here is that the past actions of MediaMarkt weren't random but rather they were a strategic response to Plaisio's actions.
- Mixed strategies is the result of different "moods" a player might have
  - A Bertrand rival acts aggressively or passively depending on her mood!
    - Do we really need to explain the problem here???

Mixed strategies: another example

The "chicken" game

Two players drive toward one another, trying to convince the other to yield and ultimately swerve into a ditch. If both swerve into the ditch, the outcome is a draw and both get zero. If one swerves and the other doesn't, the driver who swerves loses one and the other driver wins one. Note that adding a constant to a player's payoffs, or multiplying that player's payoffs by a positive constant, doesn't affect the Nash equilibria—pure or mixed. Finally, when neither yield a crash results and each player loses four. The payoff matrix is as follows:

		Player 2	
		Swerve	Straight
Player 1	Swerve	0, 0	-1, 1
	Straight	1, -1	-4, -4

- This game has two pure strategy equilibria: (Swerve, Straight) and (Straight, Swerve).
- What about a Mixed Strategy Nash Equilibrium (MSNE)? What are the expected payoffs under the MSNE?

Mixed strategies: an example

... The "chicken" game

		Player 2	
		Swerve	Straight
Player 1	Swerve	0, 0	-1, 1
	Straight	1, -1	-4, -4

The mixed strategy equilibrium is more likely, in some sense, in this game: If the players already knew who was going to yield, they wouldn't actually need to play the game. The whole point of the game is to find out who will yield, which means that it isn't known in advance. This means that the mixed strategy equilibrium is, in some sense, the more reasonable equilibrium.

Time to practice

Consider a typical "prisoner's dilemma" game as in the table below. Find the MSNE of this game.

		Player 2	
		Lie	Confess
Player 1	Lie	-1, -1	-9, 0
	Confess	0, -9	-6, -6

Practice problems

Practice with the following problems (ch. 3) from our textbook:

- Exercise 2—Lobbying Game……………………………………………………67
- Exercise 3—A Variation of the Lobbying Game ………………………..71
- Exercise 4—Mixed Strategy Equilibrium with n > 2 Players ……...73
- Exercise 5—Randomizing Over Three Available Actions …………..75
- Exercise 6—Pareto Coordination Game……………………………………78
- Exercise 7—Mixing Strategies in a Bargaining Game ………………..80