GAME THEORY

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LECTURE NOTES SET 2: COORDINATION FAILURE

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What Is Coordination Failure?

A coordination failure is....

Oh! Wait...

What Is Pareto Efficiency?

Pareto efficiency (or **Pareto optimality**) is a situation (*e.g.*, an allocation of resources) where the circumstances of an individual cannot be improved by moving to a different situation without making at least another individual worst off.

What are the Pareto Efficient states in the following examples?

Prisoner's Dilemma		Player 2	
		Lie	Confess
Player 1	Lie	-1, -1	-9, 0
	Confess	0, -9	-6, -6

Battle of Sexes		Player 2	
		UFC	Opera
Player 1	UFC	2, 1	0, 0
	Opera	0, 0	1, 2

Pareto Coordination		Player 2	
		High advert. spend.	Low advert. spend.
Player 1	High advert. spend.	5, 5	8, 2
	Low advert. spend.	2, 8	10, 10

So… What Is Coordination Failure?

A **coordination failure** is a situation where the outcome of the interaction is not Pareto efficient.

What are the Nash Eq. in the previous examples?
- What do you observe when you compare them to the Pareto efficient states?

Prisoner's Dilemma

		Player 2	
		Lie	Confess
Player 1	Lie	-1, -1	-9, 0
	Confess	0, -9	NE -6, -6

Battle of Sexes

		Player 2	
		UFC	Opera
Player 1	UFC	NE 2, 1	0, 0
	Opera	0, 0	1, 2 NE

Pareto Coordination

		Player 2	
		High advert. spend.	Low advert. spend.
Player 1	High advert. spend.	NE 5, 5	8, 2
	Low advert. spend.	2, 8	10, 10 NE

So… What Is Coordination Failure?

A **coordination failure** is a situation where the outcome of the interaction is not Pareto efficient. This happens when the **objectives of the players are not aligned**.

In decreasing order of objectives alignment, we have the following strategic situations:

1. pure coordination game

   Despite the multiple NE, players' objectives are perfectly aligned. As a result, all players will (?) eventually end up choosing their strategy that corresponds to the NE that Pareto Dominates all other NE.

2. battle of sexes game

   There are multiple NE and they are Pareto Efficient. However, none of them Pareto dominates the others: different players prefer different NE

3. Prisoner's dilemma

   The unique NE does not coincide with the Pareto Efficient allocation. As a result non-cooperative players will never reach Pareto Efficiency.

So… What Is Coordination Failure?

- the problem of coordination lies at the base of many economic situations and has been the focus of many disciplines:

  institutional approach (laws and social norms); business & management (hierarchies and organizations); public finance; economic theory; etc.

Pareto Coordination games

Coordination failure may occur in such games because of strategic uncertainty, i.e., the risk that one player chooses A while the other chooses B. However, Pareto Dominance shows that this is not, in general, a big problem (?).

Harsanyi and Selten (1988) introduced the notions of payoff and risk dominances based on the payoff structures of a game:

- A Nash equilibrium is **payoff-dominant** if it is Pareto-superior to all other Nash equilibria in a game, that is, there does not exist another equilibrium that yields greater payoffs to either player.
  - The notion of payoff dominance is based on *collective rationality*.
- A Nash equilibrium is **risk-dominant** if it is less risky compared to all other Nash equilibria in a game.
  - The notion of risk-dominance is based on the uncertainty about other players' actions.
  - How do we measure and compare risks?
    - Must calculate the product of deviation losses.

Pareto Coordination games

Example 1

Two firms produce complementary products. However, for the products to be used together there must be technologically compatible. Firms choose (simultaneously) one of two available technologies, A and B, prior to introducing the product in the market. If technologies do not match both products fail. If technologies match, profits are earned. However, both firms receive higher profits when technology A is chosen. The payoff matrix is

		Firm 2	
		Tech. A	Tech. B
Firm 1	Tech. A	10, 8	0, 0
	Tech. B	0, 0	6, 5

Evaluate the NE of this game in terms of payoff- and risk-dominance.

Pareto Coordination games

Example 1' (*assurance or stag-hunt game*)

Now consider a slight modification in the previous game. Two firms produce complementary products. However, for the products to be used together there must be technologically compatible. Firms choose (simultaneously) to introduce a new technology or to stick with the traditional one prior to introducing the product in the market. If technologies do not match the product that adopts the new tech fails while the other earns some profit. If technologies match, profits are earned. However, both firms receive higher profits when new technology A is chosen. The payoff matrix is

		Firm 2	
		Tech. A	Tech. B
Firm 1	Tech. A	10, 8	0, 4
	Tech. B	5, 0	6, 5

Evaluate the NE of this game in terms of payoff- and risk-dominance.

“Battle of Sexes” games

Coordination failure may occur in such games because of strategic uncertainty, i.e., the risk that one player chooses A while the other chooses B. Moreover, there is no Pareto Dominance in this cases.

In such games players prefer different Nash equilibria resulting in indeterminacy of the game’s outcome.

- Each pure NE is kind of “inefficient” or “unfair” in layman’s terms.
- The resulting **Mixed Strategy Nash Equilibrium** (what’s this?) is inefficient (i.e., Pareto inferior to any pure NE).
- Remedy? One possible solution is to consider a **correlated equilibrium** (what’s this?)

“Battle of Sexes” games

Example 2

Two firms produce complementary products. However, for the products to be used together there must be technologically compatible. Firm 1 currently uses tech A while firm 2 uses tech B. Firms choose (simultaneously) to stay with their current tech or switch to the other prior to introducing the product in the market. If technologies do not match both products fail. If technologies match, profits are earned. However, firm 1 receive higher profit when the match is over tech A while firm 2 earns higher profit when the match is over tech B. The payoff matrix is

		Firm 2	
		Change to A	Stay with B
Firm 1	Stay with A	3, 2	0, 0
	Change to B	0, 0	2, 4

Evaluate the pure NE of this game in terms of payoff- and risk-dominance.

Prisoner’s Dilemma games

Coordination failure occurs with certainty in such games because of individual interests are not compatible with the Pareto Efficient allocation, i.e., individually rational behaviour leads to collectively bad outcome!

In such games coordination can be achieved when a mechanism is designed so that prevents the players from deviating from the Pareto Efficient outcome (do your remember Cartels?)

- Such mechanism might require that
  - the game is played repeatedly, or
  - the payoffs change in order to make the Pareto Efficient outcome a NE (but then there is no longer a prisoner’s dilemma game!)

Coordination Failure: policy implications

In the presence of coordination failures it is important to identify the type of failure. The policy should be devised in order to tackle the specific problem:

- in the case of aligned preferences ("Pareto coordination" and "Battle of Sexes" games), we need a mechanism to facilitate cooperation
- in the case of less aligned preferences ("prisoner's dilemma" game), we need a mechanism to enforce cooperation

Problem: is it possible to identify the ex-ante objectives?

Practice problems

1. ***(Ch.2 Exercise 5—Games with Positive Externalities in our textbook)***

Two neighboring countries, $i = 1, 2$, simultaneously choose how many resources (in hours) to spend in recycling activities, r_i. The average benefit (π_i) for every dollar spent on recycling is

$$\pi_i(r_i, r_j) = 10 - r_i + \frac{r_j}{2}$$

and the (opportunity) cost per hour of recycling activity for each country is 4. Country i's average benefit is increasing in the resources that neighboring country j spends on his recycling because a clean environment produces positive external effects on other countries.

a. Find each country's best-response function, and compute the Nash Equilibrium (r_1^*, r_2^*)
b. Graph the best-response functions and indicate the pure strategy Nash Equilibrium on the graph.
c. On your previous figure, show how the equilibrium would change if the intercept of one of the countries' average benefit functions fell from 10 to some smaller number.

Practice problems

2. ***(Modifying the actions set in the previous problem)***

Two neighboring countries, $\boldsymbol{i = 1, 2}$, simultaneously choose how many resources (in hours) to spend in recycling activities, $\boldsymbol{r_i}$. However, country 2 is facing a capacity constraint: $\boldsymbol{r_2}$ cannot exceed a certain maximum, say $\boldsymbol{r_{2MAX} = 3, 4}$. The average benefit ($\boldsymbol{\pi_i}$) for every dollar spent on recycling is

$$\boldsymbol{\pi_i(r_i, r_j) = 10 - r_i + \frac{r_j}{2}}$$

and the (opportunity) cost per hour of recycling activity for each country is 4. Country $\boldsymbol{i}$'s average benefit is increasing in the resources that neighboring country $\boldsymbol{j}$ spends on his recycling because a clean environment produces positive external effects on other countries.

a. Find each country's best-response function, and compute the Nash Equilibrium $(\boldsymbol{r_1^*, r_2^*})$
b. Graph the best-response functions and indicate the pure strategy Nash Equilibrium on the graph.
c. On your previous figure, show how the equilibrium would change if the cost per hour of recycling activity increases to 5.

Practice problems

3. ***(Positive or negative externalities?)***

Two neighboring countries, $i = 1, 2$, simultaneously choose the level of recycling activities, r_i. The average benefit (π_i) on recycling is

$$\pi_i(r_i, r_j) = 10 - r_i + \frac{r_j}{2}$$

Country i's average benefit is increasing in the resources that neighboring country j spends on his recycling because a clean environment produces positive external effects on other countries. The recycling cost depends on the sum of recycling activities of the two countries and is it described by

$$C = \alpha(r_1 + r_2)^2$$

where $a \geq 0$.

a. Formally define the game.
b. Find each country's best-response function.
c. Compute the Nash equilibrium of this game.
d. How does the Nash Equilibrium change with changes in α?
e. Graph the best-response functions and indicate the pure strategy Nash Equilibrium on the graph.

Practice problems

Practice with the following problems (ch. 2) from our textbook: