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Homework on Nash Equilibrium

(20 points) **Exercise 1:** (Midterm 2017) Describe the features (options available to individuals and their preferences) of the Hawk-Dove game we studied in class. You can use a payoff matrix if you find it more convenient. Find a real-life situation that fits to this description.

(25) **Exercise 2:** (Midterm 2017) A committee of three members must decide on one of the two policy alternatives: A or B. Two of the committee members prefer that A is the chosen policy and the other one prefers B. Each of them will vote individually on either A or B. The policy alternative that receives more votes will be implemented.

(15) a) Find the *Nash equilibria* of the game described above.

NB: Recall that in a Nash equilibrium no player has a profitable deviation.

(5) b) Is there a *strictly dominant* strategy for any of the players? If yes, which one?

(5) c) Is there a *weakly dominant* strategy for any of the players? If yes, which one?

(20 points) **Exercise 3:** (Midterm 2018) Consider the following normal-form game, where the strategies for Player 1 are U, M, and D, and the strategies for Player 2 are L, C, and R. The first payoff in each cell of the matrix belongs to Player 1, and the second one belongs to Player 2.

Player 2

Player 1		L	C	R
	U	$6, 8$	$2, 6$	$8, 2$
	M	$8, 2$	$4, 4$	$9, 5$
	D	$8, 10$	$4, 6$	$6, 7$

(7) a) Find the *strictly dominated* strategies for each of the players. Make sure to write down which strategies strictly dominate them.

(7) b) Which strategies survive the process of *iterated elimination* of strictly dominated strategies?

(6) c) Find the Nash equilibria of this game.

Exercise 4: (Final 2018) The United States is deciding on the magnitude of the protectionist trade policies that it will implement. Suppose that the level of such policies can be represented by the non-negative number a_1. Simultaneously, the European Union is making a similar choice and setting its own policies $a_2 > 0$. The payoff of each player i is given by $v_i\left(a_1, a_2\right) = a_i - 2a_j + a_i a_j - \left(a_i\right)^2$.

a) Find a Nash equilibrium of this game. Is it unique?

b) Is the equilibrium outcome you found in the previous part Pareto efficient?

c) If the players could sign a binding trade agreement on the levels of protectionist policies that they could implement, what levels would they choose?

Exercise 5: (Final 2019, 30 points) *Dominated Strategies*. Consider the following Cournot duopoly game. Each of the two players will choose a non-negative real number (q_1, q_2) and the resulting payoffs will be $v_1(q_1, q_2) = (120 - q_1 - q_2) q_1$ and $v_2(q_1, q_2) = (120 - q_1 - q_2) q_2$.

(10) a) Show that strategy $q_i = 60$ strictly dominates any strategy larger than 60 for player i.

(10) b) Once you eliminate the strictly dominated strategies, you transform the game into a reduced game where players choose their strategies from set $[0, 60]$. In this reduced game, show that any value of q_i smaller than 30 is strictly dominated for player i.

(10) c) What you have showed above are the first two steps of the process of "iterated elimination of strictly dominated strategies." Suppose we know that this process will eliminate all the strategies except one. Find the strategy that would survive this iterated elimination process. You can use the property that if an iterated-elimination equilibrium exists, it is also a Nash equilibrium.

Exercise 6: (Midterm 2020, 20 points) Consider the following variation on the tragedy of commons game: Two herders simultaneously decide the number of animals to graze in the common field (a_1 and a_2). The per animal value of grazing is $200 - (a_1 + a_2)^2$. The payoff of each herder i is the total value of his herd $a_i \left[200 - (a_i + a_j)^2\right]$.

(10) a) Find the Nash equilibrium of this game. (*Hint*: you do not need to find the exact solutions for the herders' best-response functions.)

(10) b) Is the Nash equilibrium outcome *Pareto efficient*?