ECOI 31271 Game Theory

Dimitrios Zormpas

Exercise on Bayesian Games

(Final 2020, 15 points) Consider the Cournot duopoly model in which two firms, 1 and 2, simultaneously choose the quantities they supply, q_1 and q_2. The price each will face is determined by the market demand function $p(q_1, q_2) = a - b(q_1 + q_2)$. Each firm has a probability μ of having a marginal unit cost of c_L and a probability $1-\mu$ of having marginal unit cost of c_H. These probabilities are common knowledge, but the true type is revealed only to each firm individually. Solve for the *Bayesian Nash equilibrium.*

For player i with cost (type) c_i, the payoff as a function of chosen output levels:

$$v_i = (a - bq_i - bEq_j - c_i)\, q_i$$

where Eq_j is the expected output level from the other player j, that is $Eq_j = \mu q_{jL} + (1-\mu)\, q_{jH}$.

Player i chooses q_i to maximize v_i. The first order condition for this maximization is:

$$\begin{aligned} a - bq_i - bEq_j - c_i - bq_i &= 0 \\ a - 2bq_i - bEq_j - c_i &= 0 \end{aligned}$$

giving the best response function of player i with type c_i as

$$q_i = \frac{a - c_i}{2b} - \frac{1}{2}Eq_j$$

Since negative output levels are ruled out, $q_i = \max\left\{0, \frac{a-c_i}{2b} - \frac{1}{2}Eq_j\right\}$. But I will ignore this issue and assume that $\frac{a-c_i}{2b} - \frac{1}{2}Eq_j$ is positive in equilibrium (c_L and c_H are both low enough).

A Bayesian Nash Equilibrium should give us the output levels of the two types of the two agents: $q_{1L}, q_{1H}, q_{2L}, q_{2H}$. And each of these should be a best response to the output levels chosen by the other agent. This yields four equations to solve for the four unknowns.

An easy way to solve this system of equations is to take the expectation over the best response function of agent with type c_i that we found above:

$$Eq_i = \frac{a - Ec}{2b} - \frac{1}{2}Eq_j$$

where Ec is the expected cost, that is $Ec = \mu c_L + (1-\mu)\, c_H$. Using the symmetry of the players, we can see that $Eq_i = Eq_j$ and therefore

$$\begin{aligned} \frac{3}{2}Eq_i &= \frac{a - Ec}{2b} \\ Eq_i &= Eq_j = \frac{a - Ec}{3b} \end{aligned}$$

Substituting the value of Eq_j in the best-response function, we find

$$\begin{aligned}
q_i &= \frac{a-c_i}{2b} - \frac{a-Ec}{6b} \\
&= \frac{2a-3c_i+Ec}{6b} \\
&= \frac{2a-3c_i+\mu c_L+(1-\mu)\,c_H}{6b}
\end{aligned}$$

implying that

$$\begin{aligned}
q_{1L} &= q_{2L} = \frac{2a-(3-\mu)\,c_L+(1-\mu)\,c_H}{6b} \\
q_{1H} &= q_{2H} = \frac{2a-(2+\mu)\,c_H+\mu c_L}{6b}
\end{aligned}$$

Notice that, when the values of c_L and c_H are low enough, the output levels above are indeed non-negative and we have a valid Bayesian Nash equilibrium.