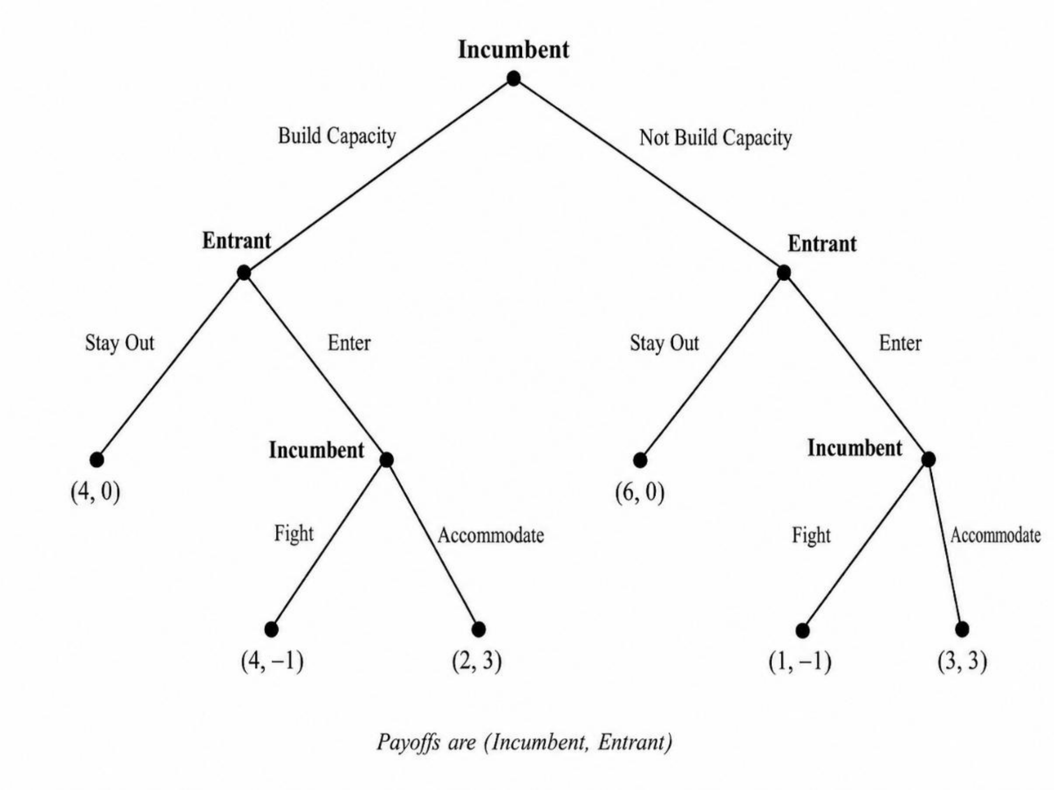
# GAME THEORY

## Assignment 2
## Suggested answers

**<u>Exercise 1:</u> Entry Deterrence with Capacity Commitment**

(a) Following what is given in the description of the situation of strategic interaction between an incumbent firm and a potential entrant, the game can be represented with the game tree below:




*Note: for answers in (b) and (c) remember that **a player's strategy must contain as many actions as the number of the nodes that belong to that player**.*

(b) Checking the game tree provided in (a), we can identify that the Entrant has two decision nodes. Hence, each strategy of the Entrant must contain two actions. Given this, the strategy set of the Entrant is

$$S_{Entrant} = \{Stay\ out\ \text{-}\ Stay\ out, Stay\ out\ \text{-}\ Enter, Enter\ \text{-}\ Stay\ out, Enter\ \text{-}\ Enter\ \}$$

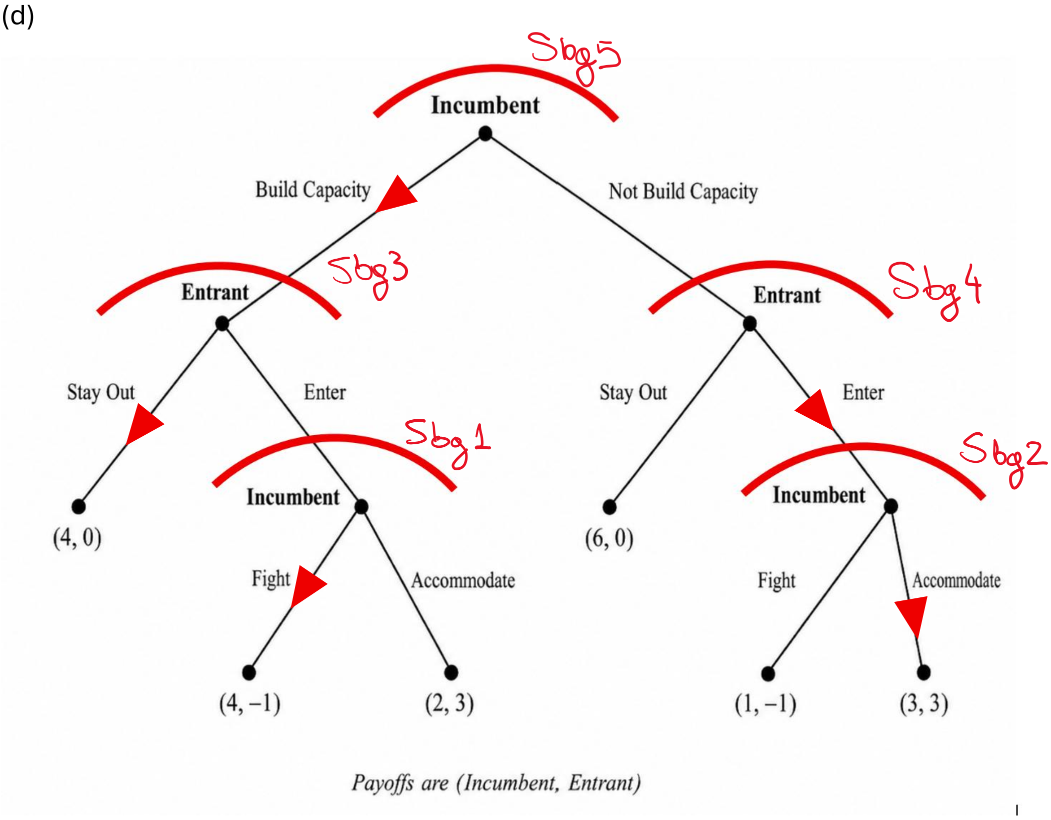
where in each strategy the first action describes what the entrant does if the incumbent firm builds capacity and the second action describes what the entrant does if the incumbent firm does not build capacity

(c) Checking the game tree provided in (a), we can identify that the Incumbent has three decision nodes. Hence, each strategy of the Incumbent must contain three actions. Given this, the strategy set of the Incumbent is

$$S_{\text{Incumbent}} =$$
$$= \{\text{Build - Fight - Fight, Build - Fight - Accommodate, Build - Accommodate - Fight,}$$
$$\text{Build - Accommodate - Accommodate, Not Build - Fight – Fight, Not Build - Fight - Accommodate,}$$
$$\text{Not Build - Accommodate - Fight, Build - Accommodate - Accommodate}\}$$

where in each strategy the first action describes if the Incumbent builds or not capacity, the second action describes what the Incumbent does if the Entrant decides to enter given that the Incumbent has built capacity, and the third action describes what the Incumbent does if the Entrant decides to enter given that the Incumbent has not built capacity.

(d)




- In Sub-game 1 (Sbg1) the Incumbent has to choose “Fight” (to receive 4) or “Accommodate” (to receive 2). He prefers to “Fight”.
- In Sub-game 2 (Sbg2) the Incumbent has to choose “Fight” (to receive 1) or “Accommodate” (to receive 3). He prefers to “Accommodate”.

- In Sub-game 3 (Sbg3) the Entrant has to choose “Stay out” (to receive 0) or “Enter”. In the latter case, the incumbent decides “Fight” (as shown in Sbg1) and the entrant receives -1. He prefers to “Stay out”.
- In Sub-game 4 (Sbg4) the Entrant has to choose “Stay out” (to receive 0) or “Enter”. In the latter case, the incumbent decides “Accommodate” (as shown in Sbg2) and the entrant receives 3. He prefers to “Enter”.
- In Sub-game 5 (Sbg5) the Incumbent has to choose “Build” (where in such case, the Entrant decides “stay out” in sbg3, so that the Incumbent receives 4) or “Not Build” (where in such case, the Entrant decides “enter” in Sbg4, and the Incumbent “accommodates” in Sbg 2, so that the Incumbent receives 3). He prefers to “Build”.

(e) Following part (d) the SPNE is

$$\text{SPNE} = \{\text{ Build - Fight - Accommodate, Stay out - Enter}\}$$

(f) Yes. Capacity investment makes the threat to fight entry credible. If the incumbent has built capacity and the entrant enters, the incumbent compares:

Fight=4 with Accommodate=2

Since 4>2 the incumbent really wants to fight after capacity has been built. Therefore, the threat to fight is credible.

Without capacity, however, the threat is not credible. If the entrant enters and the incumbent has not built capacity, the incumbent compares:

Fight=1 with Accommodate=3

Since 3>1 the incumbent would accommodate rather than fight.

Therefore, capacity investment works as a commitment device: it changes the incumbent’s future incentives so that fighting becomes optimal after entry. This is why the entrant stays out in equilibrium.

(g) In the standard entry-deterrence game, the incumbent may threaten to fight entry, but this threat is usually **not credible**. The reason is that, once entry has already occurred, fighting is costly for the incumbent. At that point, the incumbent would rather accommodate than fight. Therefore, a rational entrant ignores the threat and enters.

In this game, capacity investment changes the incumbent’s incentives. If the incumbent has built capacity, then after entry:

$$Fight = 4$$

whereas:

$$Accommodate = 2$$

So, after capacity has been built, fighting is actually better for the incumbent. The threat to fight is therefore credible.

The key difference is that capacity investment acts as a **commitment device**. It makes the incumbent's future threat believable because it changes the payoff from fighting. As a result, the entrant anticipates that entry will be met by fighting and chooses to stay out. Therefore, compared with the standard entry-deterrence game, the incumbent's threat is more convincing here because capacity investment transforms fighting from a non-credible threat into a credible strategic response.

**Exercise 2**

The stage game is described by the following payoff matrix:

| | | **Player 2** | |
|---|---|---|---|
| | | $C_2$ | $D_2$ |
| **Player 1** | $C_1$ | **4, 4** | **0, 5** |
| | $D_1$ | **6, 0** | **2, 2** |

(a) For Player 1:

- If Player 2 plays C2 , Player 1 compares choosing C1 to receive 4 with D1 to receive 6. So, Player 1 prefers D1.
- If Player 2 plays D2 , Player 1 compares choosing C1 to receive 0 with D1 to receive 2. So, Player 1 prefers D1.

Therefore, D1 is a dominant strategy for Player 1.

For Player 2:

- If Player 1 plays C1, Player 2 compares choosing C2 to receive 4 with D2 to receive 5. So, Player 2 prefers D2.
- If Player 1 plays D1, Player 2 compares choosing C2 to receive 0 with D2 to receive 2. So, Player 2 prefers D2.

Therefore, D2 is a dominant strategy for Player 2.

Hence, the unique Nash equilibrium of the stage game is NE=(D1,D2) with payoffs (2,2). Note that the outcome (C1, C2) is not a Nash equilibrium of the stage game, because each player has an incentive to deviate unilaterally: 6>4 for Player 1(moving unilaterally to D1), and 5>4 for Player 2 (moving unilaterally to D2).

(b) For each player $i = 1,2$, let $j \neq i$ denote the other player. Player $i$'s grim-trigger strategy is the following:

At $t = 0$, player $i$ plays $C_i$. For every period $t \geq 1$, player $i$ plays $C_i$ if player $j$ has played $C_j$ in every previous period. That is, player $i$ continues to cooperate as long as the other player has never defected. If player $j$ has played $D_j$ in any previous period, then player $i$ plays $D_i$ from that period onward forever. Equivalently (and more formally)

$$s_i(h^t) = C_i$$

if player $j$ has always played $C_j$ in the history $h^t$, while

$$s_i(h^t) = D_i$$

if player $j$ has played $D_j$ at least once in the history $h^t$.

Thus, player $i$ starts by cooperating and continues to cooperate as long as the other player has cooperated in the past. Once the other player defects once, player $i$ switches permanently to defection. This is why the strategy is called a **grim-trigger** strategy.

(c) **Present value from always following the grim-trigger strategy**

If both players follow the grim-trigger strategy, they cooperate forever. Each player receives 4 in every period. Therefore:

$$PV_C = 4 + \delta 4 + \delta^2 4 + \delta^3 4 + \cdots$$

Using the formula for an infinite geometric series:

$$PV_C = \frac{4}{1-\delta}$$

So, for both players

$$PV_C = \frac{4}{1-\delta}$$

**Present value from deviating once**

Suppose a player deviates in the current period while the other player cooperates. After that, punishment starts and both players play $D$ forever.

- For **Player 1**, deviating once gives 6 in the current period. From the next period onward, Player 1 receives the punishment payoff 2 forever. Therefore:

$$PV_D^1 = 6 + \delta 2 + \delta^2 2 + \delta^3 2 + \cdots$$

So, for player 1

$$PV_D^1 = 6 + \frac{2\delta}{1-\delta}$$

- For **Player 2**, deviating once gives 5 in the current period. From the next period onward, Player 2 receives the punishment payoff 2 forever. Therefore:

$$PV_D^2 = 5 + \delta 2 + \delta^2 2 + \delta^3 2 + \cdots$$

So, for player 2

$$PV_D^2 = 5 + \frac{2\delta}{1-\delta}$$

(d) For cooperation to be sustained, each player must prefer cooperation forever to deviating once and then being punished forever.

**Player 1's incentive constraint**

Player 1 cooperates if:

$$\frac{4}{1-\delta} \geq 6 + \frac{2\delta}{1-\delta}$$

Multiply both sides by $1 - \delta$:

$$4 \geq 6(1-\delta) + 2\delta$$
$$4 \geq 6 - 6\delta + 2\delta$$
$$4 \geq 6 - 4\delta$$
$$4\delta \geq 2$$
$$\delta \geq \frac{1}{2}$$

So Player 1 is willing to cooperate if $\delta \geq \frac{1}{2}$

**Player 2's incentive constraint**

Player 2 cooperates if:

$$\frac{4}{1-\delta} \geq 5 + \frac{2\delta}{1-\delta}$$

Multiply both sides by $1 - \delta$:

$$4 \geq 5(1-\delta) + 2\delta$$
$$4 \geq 5 - 5\delta + 2\delta$$
$$4 \geq 5 - 3\delta$$
$$3\delta \geq 1$$
$$\delta \geq \frac{1}{3}$$

So Player 2 is willing to cooperate if $\delta \geq \frac{1}{3}$

- Both incentive constraints must hold. Therefore:

$$\delta \geq \max\left\{\frac{1}{2}, \frac{1}{3}\right\}$$

Hence, cooperation can be sustained if $\delta \geq \frac{1}{2}$

(e) A higher discount factor means that players care more about future payoffs.

- If $\delta$ is low, players are impatient. The short-run gain from deviation is important, while the future punishment is not very costly. Therefore, cooperation is difficult to sustain.
- If $\delta$ is high, players are patient. They care more about the future loss caused by punishment. In that case, the one-period gain from deviation is not worth losing the future benefits of cooperation.

In this game, Player 1 has the stronger temptation to deviate, because Player 1 can increase the current payoff from $4$ to $6$. Player 2 can increase the current payoff from $4$ to $5$. Therefore, Player 1's incentive constraint is stricter.
That is why the relevant threshold is

$$\delta \geq \frac{1}{2}$$

When $\delta \geq \frac{1}{2}$, grim-trigger strategies can sustain cooperation.