GAME THEORY

Assignment 2

Instructions

You must answer both questions of this assignment. Each question carries equal weight. This is an individual project and accounts for 10% of your final grade on this course. The project is due on 05/06/2026 and you will have to submit your answer online (by sending them to efilipp@uom.edu.gr). Late assignments WON'T be accepted.

Exercise 1: Entry Deterrence with Capacity Commitment

A market is initially served by an incumbent firm. Before a potential entrant decides to enter the market, the incumbent can make an observable investment in capacity.

The timing of the game is as follows:

- The incumbent first chooses whether to **Build Capacity** or **Not Build Capacity**.
- The entrant observes the incumbent's capacity decision and then chooses whether to **Enter** or **Stay Out**.
    - If the entrant enters, the incumbent observes entry and chooses whether to **Fight** or **Accommodate**.
    - If the entrant stays out, the game ends immediately.

The payoffs are as follows. The first payoff belongs to the entrant and the second payoff belongs to the incumbent.

- If the incumbent builds capacity and the entrant stays out, the entrant receives 0 and the incumbent receives 4.
- If the incumbent builds capacity, the entrant enters, and the incumbent fights, the entrant receives −1 and the incumbent receives 4.
- If the incumbent builds capacity, the entrant enters, and the incumbent accommodates, the entrant receives 3 and the incumbent receives 2.
- If the incumbent does not build capacity and the entrant stays out, the entrant receives 0 and the incumbent receives 6.
- If the incumbent does not build capacity, the entrant enters, and the incumbent fights, the entrant receives −1 and the incumbent receives 1.
- If the incumbent does not build capacity, the entrant enters, and the incumbent accommodates, the entrant receives 3 and the incumbent receives 3.

a. Draw the extensive-form game tree.
b. List the strategy set of the entrant.
c. List the strategy set of the incumbent.
d. Solve the game by backward induction.
e. Find the Subgame Perfect Nash Equilibrium.
f. Explain whether capacity investment makes the threat to fight entry credible.
g. Compare this game with the standard entry-deterrence game discussed in class. Why is the incumbent's threat to fight entry more convincing after capacity has been built?

Exercise 2

Consider the following stage game:

		Player 2	
		C_2	D_2
Player 1	C_1	4, 4	0, 5
	D_1	6, 0	2, 2

The game is repeated infinitely many times. Both players observe all past actions. Let $\delta \in (0,1)$ be the common discount factor. Each player maximizes the present value of payoffs:

$$PVU_i = u_{i,0} + \delta u_{i,1} + \delta^2 u_{i,2} + \delta^3 u_{i,3} + \cdots$$

Consider the following grim-trigger strategy: each player starts by playing C, continues to play C as long as both players have always played C in the past, and plays D forever after any player has played D once.

a. Find the Nash equilibrium of the stage game. Is (C,C) a Nash equilibrium of the stage game?
b. Write the grim-trigger strategy carefully as a complete contingent plan.
c. For each player, calculate:
   - the present value from always following the grim-trigger strategy;
   - the present value from deviating once while the other player continues to cooperate, followed by punishment forever.
d. Derive the minimum value of δ for which cooperation can be sustained by grim-trigger strategies.
e. Explain the economic intuition. Why does a higher δ make cooperation easier to sustain?