

GAME THEORY

Assignment 1

Instructions

You must answer all questions of this assignment. Each question carries equal weight. This is an individual project and accounts for 10% of your final grade on this course. The project is due on 30/03/2026 in class. Late assignments WON'T be accepted.

Exercise 1

Last helicopter ride from a remote research station

Two researchers at a remote field station learn that a helicopter is leaving because of an incoming storm. The pilot explains that he can take one researcher under normal flying conditions, together with that researcher's essential luggage, files, data drives, samples, and research equipment. He can also take both researchers, but only if they travel with minimal weight, which means leaving behind most of their luggage and, crucially, a substantial part of their research material (such as field notes, samples, storage devices, or experimental equipment). Each researcher must decide independently and simultaneously whether to board.

Each researcher ranks outcomes as follows:

- Best (payoff 3): leave alone. In that case, the researcher can evacuate safely and take along the essential products of their work.
 - Second best (payoff 2): leave together. Both escape the storm, but each must abandon valuable research materials, reducing the benefit of departure.
 - Worst (payoff 1): remain at the station waiting for another flight, facing danger, delay, and uncertainty.
- a. Fully describe the game and represent the situation with the use of a payoff matrix.
 - b. Is there any strictly dominated strategy for player 1? Is there any strictly dominated strategy for player 2? Explain your answer.
 - c. What is (are) the Nash Equilibrium (Equilibria) in pure strategies? Explain your answer.
 - d. Is there a coordination failure in this game? Explain your answer.
 - e. Let each player assign a probability distribution over her actions. What is the Nash Equilibrium in Mixed Strategies? Explain your answer.
 - f. Given you answer in part (e), do you want to change your answer in part (b)?

Exercise 2

Consider a static game between two players as described by the payoff matrix below:

		Player 2		
		Left	Center	Right
Player 1	Up	2, 2	0, 1	2, 0
	Middle	3, 0	1, 1	0, 1
	Down	0, 2	1, 0	3, 3

- What are the actions/strategies sets for the two players?
- Is there any strictly dominated strategy for player 1? Is there any strictly dominated strategy for player 2? Explain your answer.
- What is (are) the Nash Equilibrium (Equilibria) in pure strategies? Explain your answer.
- Is there a coordination failure in this game? Explain your answer.
- Let each player assign a probability distribution over her actions. What is the Nash Equilibrium in Mixed Strategies? Explain your answer.
- Given you answer in part (e), do you want to change your answer in part (b)?

Exercise 3

Voluntary Contribution to a Public Good

Consider a group of n individuals, indexed by $i = 1, 2, \dots, n$. Each individual must simultaneously decide whether to contribute to the provision of a public good. Contributing entails a private cost $c > 0$. The public good is provided if at least one individual contributes. If the public good is provided, every individual receives a benefit $b > 0$, regardless of whether they contributed or not. Assume that $b > c > 0$. Thus, provision of the public good is beneficial, but contributing is personally costly.

Let m denote the number of contributors in the group. The payoff of player i is given as follows:

- if $m = 0$, no public good is provided and each player receives payoff 0;
 - if $m \geq 1$, the public good is provided:
 - each contributor receives $b - c$,
 - each non-contributor receives b .
- Describe this situation as a strategic-form game. Clearly specify the set of players, the strategy set of each player, the payoff function.
 - Does any player have a strictly dominant strategy?
 - Find all Nash Equilibria of this game in pure strategies.