Lecture 9

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Exercise

- Consider the following games:

$v1/v2$	B	S
B	4, 2	1, 1
S	0, 2	2, 1

$v1/v2$	B	S
B	5, 2	1, 1
S	6, 1	2, 2

- Suppose each player knows which game is played. Find the N.E.
- Suppose none knows which game is played. The former game is played with probability 1/3 whereas the latter with probability 2/3. Find the N.E.
- Suppose both players know that the payoffs of P1 are as in first game whereas P1 does not know if the payoffs of P2 are as in the first or second game. Find the BNE.

Exercise

- Suppose each player knows which game is played. Find the N.E.
  - We have BB for the first and SS for the second.
- Suppose none knows which game is played. Find the N.E.
  - The payoffs in this case are as follows:

$v1/v2$	B	S
B	$4\frac{1}{3}+5\frac{2}{3}, 2\frac{1}{3}+2\frac{2}{3}$	$1\frac{1}{3}+1\frac{2}{3}, 1\frac{1}{3}+1\frac{2}{3}$
S	$0\frac{1}{3}+6\frac{2}{3}, 2\frac{1}{3}+1\frac{2}{3}$	$2\frac{1}{3}+2\frac{2}{3}, 1\frac{1}{3}+2\frac{2}{3}$

Or

$v1/v2$	B	S
B	$14/3, 2$	$1, 1$
S	$4, 4/3$	$2, 5/3$

- BB and SS are the NE. In mixed, the column (row) can choose $q = 3/5$ ($p = 1/4$) and make the row (column) indifferent.

Exercise

- The tables are as follows:

$v1/v2$	B	S
B	$4, 2$	$1, 1$
S	$0, 2$	$2, 1$

$v1/v2$	B	S
B	$5, 2$	$1, 1$
S	$6, 2$	$2, 1$

- We start with the single type player
- Suppose that the column plays B for sure:
  - The row of type a (b) chooses B (S) since 4>0 (6>5)
  - The column does not have a profitable deviation if

$$\begin{aligned} Ev_{\text{column}}(B) &\geq Ev_{\text{column}}(S) \\ 2\frac{1}{3}+2\frac{2}{3} &\geq 1\frac{1}{3}+1\frac{2}{3} \\ 2 &\geq 1 \end{aligned}$$

Exercise

- B, BS is a BNE
- Suppose that the column plays S for sure
  - The row of type a (b) chooses S (S) since 2>1 (2>1)
  - The column does not have a profitable deviation if

$$\begin{aligned} Ev_{\text{column}}(S) &\geq Ev_{\text{column}}(B) \\ 1\frac{1}{3} + 1\frac{2}{3} &\geq 2\frac{1}{3} + 2\frac{2}{3} \\ 1 &\geq 2 \end{aligned}$$

- S, SB is not a BNE (obvious)
- HW: Suppose both players know that the payoffs of P1 are as in the second game. Then do the same for P2.

PBNE

- tree1...
- Continuation game:
  - A continuation game consists of an information set (which is not necessarily a singleton information set) and all subsequent nodes.
  - Here there are two continuation games: the whole game and the continuation game consisting of the non-singleton information set I and the following nodes
  - There is only one subgame though...
  - Rationality in the continuation game is tested

PBNE

- Rationality: We need to analyze P2's **beliefs** regarding the node where she is at.
- At each information set, the player who moves at that information set has **beliefs** over the set of nodes in that information set. This will be captured by a distribution of probabilities over the set of nodes in the information set.
- At each information set, strategies must be optimal, given the beliefs and subsequent strategies.

PBNE

- Beliefs of P2 in information set I: μ $(1-\mu)$ the probability assigned to the node following T (B)
- Given these beliefs P2's expected payoffs are $1\mu + 2(1-\mu) = 2-\mu$ when playing L and $0\mu + 1(1-\mu) = 1-\mu$ when playing R.
- Because $2-\mu > 1-\mu$, the subgame perfect equilibrium (OR) does not pass the rationality test.
- Q: How to form beliefs?

PBNE

- Still assuming that the beliefs of P2 in information set I are μ and $1-\mu$ regarding the node following T (B).
- Now suppose that P1 is playing O, T and B with probabilities $\beta_1(O), \beta_1(T)$ and $\beta_1(B)$
- If $\beta_1(T)=1$ is $\mu=0$ a consistent belief?
- No, the only consistent belief is $\mu=1$.
- Player 2 receives a signal from Player 1 when he has to make a choice at the information set I (signaling games).
- Player 1 is called Sender and Player 2 Receiver.
- We will apply Bayes' rule (whenever possible) to achieve consistency:

$$\mu=\frac{\beta_1(T)}{\beta_1(T)+\beta_1(B)} \text{ given } \beta_1(T)+\beta_1(B)>0$$

PBNE

- ...this is just a consistency condition.
- Definitely not the strictest.
- A PBNE is an assessment (μ, β) which satisfies
  1. beliefs at each information set
  2. sequential rationality
  3. Bayesian consistency

PBNE

- Let's solve the game (tree 2).
- $\mu, 1-\mu$ beliefs for P2
- $\beta_1(O), \beta_1(T), \beta_1(B)$ and $\beta_2(R), \beta_2(L)$ probabilities
- Suppose that P2 plays L for sure ($\beta_2(L) = 1$).
- Sequential rationality implies:

$$\begin{aligned} 1\mu + 1(1-\mu) &\geq 0\mu + 2(1-\mu) \\ \mu &\geq \frac{1}{2} \end{aligned}$$

- If P2 plays L for sure, P1 should choose T (2>1>0): $\beta_1(T) = 1$
- Is $\mu \geq \frac{1}{2}$ consistent with this behavior?

$$\mu = \frac{\beta_1(T)}{\beta_1(T) + \beta_1(B)} = \frac{1}{1+0} = 1$$

- $\beta_2(L) = 1, \beta_1(T) = 1$ and $\mu = 1$ is a PBNE.

PBNE

- Suppose that P2 plays R for sure ($\beta_2(R) = 1$).
- Sequential rationality implies:

$$\begin{aligned} 1\mu + 1(1-\mu) &\leq 0\mu + 2(1-\mu) \\ \mu &\leq \frac{1}{2} \end{aligned}$$

- If P2 plays R for sure, P1 should choose O (1>0): $\beta_1(O) = 1 \rightarrow \beta_1(T) + \beta_1(B) = 0$
- Bayes rule cannot be applied
- Continuum of PNBE with $\beta_2(R) = 1, \beta_1(O) = 1$ and $\mu \leq \frac{1}{2}$.

PBNE

- Suppose that P2 is indifferent between R and L.
- Sequential rationality implies:

$$\begin{aligned} 1\mu + 1(1-\mu) &= 0\mu + 2(1-\mu) \\ \mu &= \frac{1}{2} \end{aligned}$$

- P1 can choose O and get 1, T and get $2\beta_2(L) + 0\beta_2(R) = 2\beta_2(L)$ or B and get $0\beta_2(L) + 0\beta_2(R) = 0$
- Hence $\beta_1(B) = 0$

PBNE

P1 plays O ($\beta_1(O) = 1$) if $1 \geq 2\beta_2(L) \rightarrow \frac{1}{2} \geq \beta_2(L)$ and T otherwise.

1. If $\beta_1(O) = 1$ Bayes' rule is not applicable: $\beta_1(O) = 1, \frac{1}{2} \geq \beta_2(L)$ and $\mu = \frac{1}{2}$ is PBNE.
2. If $\beta_1(T) = 1$ then $\mu = \frac{\beta_1(T)}{\beta_1(T)+\beta_1(B)} = \frac{1}{1+0} = 1$ which contradicts $\mu = \frac{1}{2}$. No PBNE here.
3. If $\beta_1(O) \in (0,1)$ then $\mu = \frac{\beta_1(T)}{\beta_1(T)+\beta_1(B)} = \frac{\beta_1(T)}{\beta_1(T)+0} = 1$ which contradicts $\mu = \frac{1}{2}$. No PBNE here.

PBNE

- There are three kinds of equilibria:
  - $\beta_2(L) = 1, \beta_1(T) = 1$ and $\mu = 1$
  - $\beta_2(R) = 1, \beta_1(O) = 1$ and $\mu \leq \frac{1}{2}$
  - $\beta_1(O) = 1, \frac{1}{2} \geq \beta_2(L)$ and $\mu = \frac{1}{2}$

Cho Kreps (1987)

- Consider the following two-player signaling game.
- There are two types of Player 1, real man and wimp, which Nature chooses with probability 0.9 and 0.1 respectively.
- Both players know this probability but only Player 1 observes Nature's move.
- Player 1 is sitting in a bar when Player 2 walks in. Player 2 is the rowdy type and wants to pick a fight.
- But since he is also a coward, he only wants to fight a wimp.
- Player 2 has two actions, fight (F) and not fight (N) but before he makes the choice, he observes Player 1's behaviour.
- Player 1 can choose to drink beer (B) or eat quiche (Q) and obtains one extra unit of utility if he consumes his most preferred breakfast.
- Real men prefer beer and wimps prefer quiche.
- Player 2's payoff does not depend on player 1's breakfast and is 1 if it fights the wimp or if he avoids fighting the real man, and zero otherwise.

The end

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