Lecture 8

Dimitrios Zormpas

CY Cergy Paris Universitè

March 7th, 2022

Overview

- Prisoner's dilemma with two types of P2 (using the steps)
- Example no3
- PBNE
- Miscellanea

Overview

- **Prisoner's dilemma with two types of P2 (using the steps)**
- Example no3
- PBNE
- Miscellanea

Return to prisoner's dilemma

- P2 can be of type a:

$v1/v2$	D	C
D	$0, -2$	$-5, -1$
C	$-1, -5$	$-4, -4$

- or of type b:

$v1/v2$	D	C
D	$0, -2$	$-5, -3$
C	$-1, -5$	$-4, -6$

Return to prisoner's dilemma (cont.)

- Player 2 type b receives an additional jail term of 2 years in case that she confesses.
- Player 2 knows her type. But player 1 does not know it. Player 1 assigns the probability p to the possibility that Player 2 is type a.
- To talk about the equilibrium, we need to specify how the different types of different players will play the game.

Return to prisoner's dilemma (cont.)

- Easy to see that Player 2 type a plays confess and type b plays deny.
- What about Player 1?
- $Ev(deny) = p(-5) + (1-p)0 = -5p$
- $Ev(confess) = p(-4) + (1-p)(-1) = -3p - 1$
- If $p > 1/2$ then Player 1 plays confess.
- If $p < 1/2$ then Player 1 plays deny

Return to prisoner's dilemma (cont.)

1. Start checking the strategies of the one-type player.
2. We are looking for BRs (there should be no profitable deviation).
3. Repeat until all the strategies are checked.
4. If a strategy is a BR under some conditions, then under these conditions it gives us a BNE.
5. If a strategy is a BR under no conditions, then it cannot be chosen in a BNE.

Return to prisoner's dilemma (cont.)

- NB: We have a one-type row player and a two-type column player.
- Step 1: We start checking under what conditions the strategies of the one-type player are BRs:
  - Suppose that under certain conditions (to be identified) my BR is D.
    - In this case, type a of column player chooses C (-1>-2)
    - In this case, type b of column player chooses D (-2>-3)
  - For my BR to be indeed D I need the expected payoff to be bigger or equal to the payoff from deviating:

$$\begin{aligned} -5p + 0(1-p) &\geq -4p + (-1)(1-p) \\ -5p &\geq -3p - 1 \\ \frac{1}{2} &\geq p \end{aligned}$$

- So, under $1/2 \geq p$, D is a BR for the row player and C (D) is a BR for a column player of type a (b)
- Under $1/2 \geq p$, $\{D, CD\}$ is a BNE

Return to prisoner's dilemma (cont.)

- Under certain conditions D is a BR of the row player.
- Let's now check under what conditions (if any) C is a BR of the row player.
- Suppose that under certain conditions (to be identified) my BR is C.
  - In this case, type a of column player chooses C (-4>-5)
  - In this case, type b of column player chooses D (-5>-6)
  - For my BR to be indeed C I need the expected payoff to be bigger or equal to the payoff from deviating:

$$\begin{aligned} -4p + (-1)(1-p) &\geq (-5)p + 0(1-p) \\ -1 &\geq -2p \\ \frac{1}{2} &\leq p \end{aligned}$$

- So, under $1/2 \leq p$, C is a BR for the row player and C (D) is a BR for a column player of type a (b)
- Under $1/2 \leq p$, $\{C, CD\}$ is a BNE

Overview

- Prisoner's dilemma with two types of P2 (using the steps)
- **Example no3**
- PBNE
- Miscellanea

Choosing a movie

- row player can be of type a:

$v1/v2$	S	B
S	2, 1	0, 0
B	0, 0	1, 2

- or of type b:

$v1/v2$	S	B
S	1, 1	−1, 0
B	0, 0	1, 2

Again...

1. Start checking the strategies of the one-type player.
2. We are looking for BRs (there should be no profitable deviation).
3. Repeat until all the strategies are checked.
4. If a strategy is a BR under some conditions, then under these conditions it gives us a BNE.
5. If a strategy is a BR under no conditions, then it cannot be chosen in a BNE.

Choosing a movie (cont.)

- NB: We have a one-type column player and a two-type row player.
- Step 1: We start checking under what conditions the strategies of the one-type player are BRs:
  - Suppose that under certain conditions (to be identified) my BR is S.
    - In this case, type a of row player chooses S (2>0)
    - In this case, type b of row player chooses S (1>0)
  - For my BR to be indeed S I need the expected payoff to be bigger or equal to the payoff from deviating:

$$\begin{aligned} 1p + 1(1-p) &\geq 0p + 0(1-p) \\ 1 &\geq 0 \end{aligned}$$

- So, for any p, S is a BR for the column player and S is a BR for the row player no matter the type: $\{S, SS\}$

Choosing a movie (cont.)

- Let's now check under what conditions (if any) B is a BR of the column player.
- In this case, type a of row player chooses B (1>0)
- In this case, type b of row player chooses B (1>-1)
- For my BR to be indeed B I need the expected payoff to be bigger or equal to the payoff from deviating:

$$\begin{aligned} 2p + 2(1-p) &\geq 0p + 0(1-p) \\ 2 &\geq 0 \end{aligned}$$

- So, for any p, B is a BR for the column player and B is a BR for the row player no matter the type: $\{B, BB\}$

Choosing a movie (cont.)

- Let us discuss the steps.
- Find the N.E. in the following:

$v1/v2$	S	B
S	2, 1	0, 0
B	0, 0	1, 2

- SS and BB are the N.E.
- Let's use the steps:
- Suppose that (under certain conditions) the row player plays S (B).
- Then the column player chooses S (B).
- The row does not have an incentive to deviate since 2>0 (1>0) so SS (BB) is a N.E.
- We basically follow the same steps...

Overview

- Prisoner's dilemma with two types of P2 (using the steps)
- Example no3
- **PBNE**
- Miscellanea

Back to sequential games

- For sequential games we discussed the notion of SPNE
- Non-credible threats are filtered out
- There is a problem however when there are non-singleton information sets!
- SPNE tests rationality of the players in every subgame.
- But does not test rationality of the players at non-singleton information sets.
- Example...

Normal form

- Each player has one information set so for player 1 we have $\{O, T, B\}$ and for player 2 we have $\{L, R\}$:

$v1/v2$	L	R
O	$1,3$	$1,3$
T	$2,1$	$0,0$
B	$0,2$	$0,1$

- 2 N.E. $\{TL, OR\}$
- Since we have one subgame, they are both SPNE.
- But OR is problematic since R is strictly dominated by L in the information set...
- SPNE is not adequate when there are non-singleton information sets
- PBNE: Hint; P2 does not know if P1 plays T/B but does know that she does not play O!

Overview

- Prisoner's dilemma with two types of P2 (using the steps)
- Example no3
- PBNE
- **Miscellanea**

Miscellanea (incomplete/imperfect)

- An incomplete information game (uncertainty about the payoffs like the "leaving/loving" variation of the battle of the sexes) expressed as an imperfect information game (one player does not know the complete history of the game).
- There are mixed strategy BNEs

Miscellanea (commitment)

- Army 1, of country 1, must decide whether to attack army 2, of country 2, which is occupying an island between the two countries. In the event of an attack, army 2 may fight, or retreat over a bridge to its mainland. Each army prefers to occupy the island than not to occupy it; a fight is the worst outcome for both armies.
- Model this situation as an extensive game.
- Show that army 2 can increase its subgame perfect equilibrium payoff (and thus reduce army 1's payoff) by burning the bridge to its mainland (an act which is assumed to entail no cost), eliminating its option to retreat if attacked.

Miscellanea (OOAs)

- Firms maximize their own value (Smith, 1776).
- Reasonable if
  - i) large shareholders do not have significant shareholdings in other firms
  - ii) diversified minority shareholders have no significant influence on corporate strategy

Miscellanea (OOAs)

- Present day ownership structures look different
- examples from[1]
  - airlines
  - banks
  - supermarkets
  - pharmaceuticals
  - see companies
- Similar phenomenon outside of the USA
- Tables...

[1] Schmalz, 2018; Banal Estanol et al., 2021; Torshizi and Clapp, 2021

Miscellanea (OOAs)

- **Main idea:** "When my shareholders also hold large shares in competitors then I should not maximize the value of my firm but the value of the portfolio of my shareholders."
- **Concern:** Internalization of rival values will lead to anticompetitive behavior.
- Up to now focus on product market effects: "Are product prices higher/firm capacities lower?"
- repeat the Cournot/Bertrand/Stackelberg games we have seem where a firm is not maximizing profits but $\pi_i + \lambda\pi_j$ where $\lambda \in (0,1)$ is the so-called degree of internalization.
- What do you find? Study the effect of λ.

Summary

- Prisoner's dilemma with two types of P2 (using the steps)
- Example no3
- PBNE
- Miscellanea

The end

dimitrios.zormpas@cyu.fr